Q1)

$$E = \frac{1}{i\omega\mu_0\epsilon_0}\nabla(\nabla\cdot A) - i\omega A \quad \text{eq. } 9.21$$

$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$A = \frac{\mu_0 I_0 d e^{-ikr}}{4\pi r}\hat{z} \quad \text{eq. } 9.25$$

$$\Rightarrow A_r = \frac{\mu_0 I_0 d e^{-ikr}}{4\pi r}\cos\theta; \quad A_\theta = \frac{-\mu_0 I_0 d e^{-ikr}}{4\pi r}\sin\theta$$

$$\nabla\cdot A = \frac{1}{r^2}\frac{\delta(r^2 A_r)}{\delta r} + \frac{1}{r\sin\theta}\frac{\delta(A_\theta \sin\theta)}{\delta\theta}$$

$$= \frac{1}{r^2}\frac{\delta}{\delta r}\left(\frac{r^2\mu_0 I_0 d e^{-ikr}}{4\pi r}\cos\theta\right) + \frac{1}{r\sin\theta}\frac{\delta}{\delta\theta}\left(-\frac{\mu_0 I_0 d e^{-ikr}}{4\pi r}\sin^2\theta\right)$$

$$= \frac{\mu_0 I_0 d\cos\theta}{4\pi r^2}\frac{\delta}{\delta r}(r e^{-ikr}) - \frac{\mu_0 I_0 d e^{-ikr}}{4\pi r^2\sin\theta}\frac{\delta}{\delta\theta}(\sin^2\theta)$$

$$\frac{\delta}{\delta r}(r e^{-ikr}) \to e^{-ikr}(1-ikr) \qquad \frac{\delta}{\delta\theta}(\sin^2\theta) \to 2\sin\theta\cos\theta$$

$$=$$

$$\nabla\cdot A \Rightarrow = -\frac{ik\mu_0 I_0 e^{-ikr} d}{4\pi r}\cos\theta$$

$$\nabla(\nabla\cdot A)$$

$$= \nabla\left(-\frac{ik\mu_0 I_0 e^{-ikr} d}{4\pi r}\cos\theta\right)$$

$$= \frac{\delta}{\delta r}(\nabla\cdot A)\hat{r} + \frac{1}{r}\frac{\delta(\nabla\cdot A)}{\delta\theta}\hat{\theta} \to \frac{1}{r^2} \to 0 \text{ at } r\to\infty$$

$$\Rightarrow$$

$$\frac{-ik\mu_0 I_0 e^{-ikr}\cos\theta}{4\pi r}\,\frac{e^{-ikr}}{r}\left(-ik - \cancel{1/r}\right)\cdot\hat{r}$$

$$\frac{-ik\mu_0 I_0 e^{-ikr}\cos\theta}{4\pi r}\; \frac{e^{-ikr}}{r}\Big(\underbrace{-ik - 1/r}_{0\ \ r\to\infty}\Big)\cdot\hat{r}$$

$$\Rightarrow \quad \frac{k^2 \mu_0 I_0 d e^{ikr}}{4\pi r^2}\cos\theta$$

from 9.2)

$$E = \frac{1}{i\omega\mu_0\varepsilon_0}\;\frac{k^2\mu_0 I d e^{-ikr}}{4\pi r^2}\cos\theta\,\hat{r} + \underbrace{\frac{i\omega I_0 d e^{-ikr}\mu_0}{4\pi r}\sin\theta\,\hat{\theta}}_{\text{dominates far field}}$$

Q2) $S = P/4\pi r^2$ power density

$P = 1000$

$r = 1000$ m

$$S = \frac{1000}{4\times 3.14\times 10^6} \approx \underline{7.96\times 10^{-5}}\ \text{W/m}^2$$

$$S = \frac{E_0^2}{\eta} \qquad \eta = \sqrt{\mu_0/\varepsilon_0} \approx 377\,\Omega$$

$$E_0 = \sqrt{S\eta}$$

$$\approx \underline{0.173}\ \text{V/m}$$

Q3) $R_{load} = R_{rad}$

$$R_{rad} = \frac{2\pi}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{d}{\lambda}\right)^2 \qquad d = \lambda/2$$

$$\Rightarrow R_{rad} \approx \underline{73\,\Omega}$$

Q4) $g = 1.5$

$A = 3\lambda^2$

$$A = \frac{3\lambda^2}{8\pi}$$

$$A/g = \frac{\lambda^2}{4\pi}$$