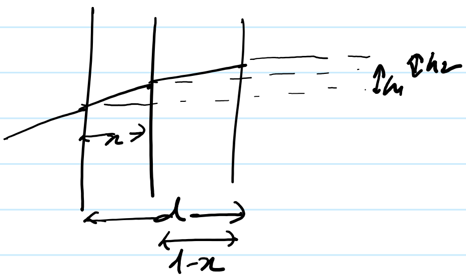
PS7

Monday, March 31, 2025 10:28 PM

Q1) total travel time $= \dfrac{\text{distance medium 1}}{\text{speed m1}} + \dfrac{\text{distance m2}}{\text{speed m2}}$



$$= \frac{\sqrt{x^2+h_1^2}}{v_1} + \frac{\sqrt{(d-x)^2+h_2^2}}{v_2}$$

$v = c/n$

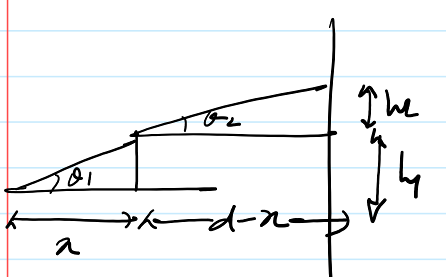
$$\Rightarrow t = \frac{n_1}{c}\sqrt{x^2+h_1^2} + \frac{n_2}{c}\sqrt{(d-x)^2+h_2^2}$$

minimize

$\dfrac{dt}{dx} = 0$ $\Rightarrow$ $\dfrac{d}{dx}\left(\dfrac{n_1}{c}\sqrt{x^2+h_1^2} + \dfrac{n_2}{c}\sqrt{(d-x)^2+h_2^2}\right)$

~~minimizing wrt h~~

$$= \frac{n_1}{c}\frac{h_1}{\sqrt{x^2+h_1^2}} - \frac{n_2}{c}\frac{h_2}{\sqrt{(d-x)^2+h_2^2}} = 0$$



$$\Rightarrow \frac{n_2}{n_1} = \frac{h_1}{\sqrt{x^2+h_1^2}} \times \frac{\sqrt{(d-x)^2+h_2^2}}{h_2}$$

$$= \frac{\sin\theta_1}{\sin\theta_2}$$

Q2) a) $R = \dfrac{P_{\text{reflected}}}{P_{\text{incident}}}$ $\quad T = \dfrac{P_{\text{transmitted}}}{P_{\text{incident}}}$

$S = E \times H$

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$$\text{Avg} = \frac{1}{2} E \times H^{*} = \frac{1}{2}\frac{E^2}{\eta} \qquad \eta = \frac{\mu_0 c}{n}$$

$$(S)$$

for $\perp$ s-polarized light

$$r_s = E_r / E_i = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

$$t_s = E_t / E_i \quad \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

for // p-polarized light

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = E_r / E_i$$

$$t_p = \frac{2 n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = E_t / E_i$$

$$S_{\text{incident}} = \frac{E_i^2}{2\eta_1} \qquad S_{\text{ref}} = \frac{E_r^2}{2\eta_1} \qquad S_t = \frac{E_r^2}{2\eta_2}$$

$$R = \frac{S_r}{S_i} = \frac{E_r^2}{E_i^2} \qquad T = 1 - R$$

simple.

$$(r_s)^2 \qquad\qquad (r_p)^2$$

b) at normal incidence

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = (0.2)^2 = 0.04$$

c) $\theta_B = \tan^{-1}(n_2/n_1)$ no polarized light is reflected.

$$\theta_B = \tan^{-1}(1/1.5)$$

$$\approx 33.7^{\circ}$$

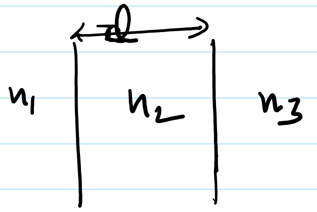
d) $\angle$ above which total internal refln. occurs.

$$\theta_c = \sin^{-1}(n_2/n_1)$$

$$= \sin^{-1}(1/1.5) \approx 42^{\circ}$$

Q3)

a)



$$r_{ij} = \frac{n_i - n_j}{n_i + n_j}$$

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} \qquad r_{23} = \frac{n_2 - n_3}{n_2 + n_3}$$

$$\phi = \frac{2\pi}{\lambda}(2d\,n_2) = \frac{4\pi n_2 d}{\lambda}$$

$$r_{total} = r_{12} + \frac{r_{23}e^{i\phi}}{1 + r_{12}r_{23}e^{i\phi}}$$

b) vanishing reflection $\longrightarrow$ destructive interference

$$\Rightarrow \phi = \frac{4\pi n_2 d}{\lambda} = (2m+1)\pi$$

$$d = \frac{(2m+1)\lambda}{4n_2}$$

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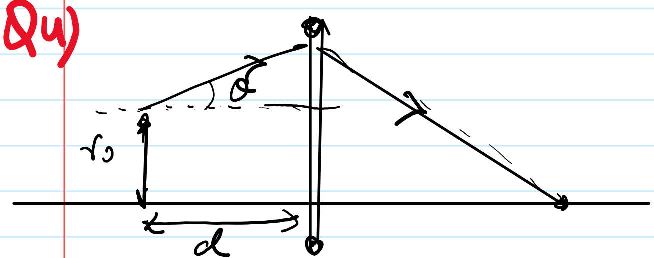
$m=0$ for min thickness $\Rightarrow$ $d = \lambda/4n_2$.

for perfect non refl $\rightarrow$ $n_2 = \sqrt{n_1 n_3}$

Q4)



$$M_{free}(d_1) = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}$$

$$M_{lens}(f) = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_{total} = M_{lens} \cdot M_{free}$$

$$= \begin{bmatrix} +1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d_1 \\ -1/f & 1-d_1/f \end{bmatrix}$$

$$M_{free}(d_2) = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}$$

$$M_{img} = M_{free\,d_2} \cdot M_{total} = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ -1/f & 1-d_1/f \end{bmatrix}$$

$$= \begin{bmatrix} 1-d_2/f & d_1 + d_2(1-d_1/f) \\ -1/f & 1-d_1/f \end{bmatrix}$$

independent of θ.

$$\begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix} = M_{img} \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix} \longrightarrow$$ r_3 should be independent of θ_0

$$\Rightarrow \quad d_1 + d_2(1 - d_1/f) = 0$$

$$\Rightarrow \quad 1/d_1 + 1/d_2 = f$$

$$M = r_3/r_0 = 1 - d_2/f$$

$$M = \frac{f}{f - d_1}$$

Q5)

a) $\theta = \lambda/D$ diffraction limit

$\lambda = 790\,nm \qquad D = 1\,\mu m$

$$\theta \approx \frac{790\times10^{-9}}{10^{-6}} \approx 0.79 \text{ rad}$$

$$\approx 45^0$$

b) $\lambda' = \theta D'$

$D' = 0.1\,\mu m$

$\theta = 0.79$

$$\lambda' = 79\,nm$$

c) $\theta = 1.22 \dfrac{\lambda}{D}$ rayleigh criterion

min. resolvable $\angle$.

$\lambda = 600\,nm$

letter: $L = 0.1\,m$

letter $h = 0.1\,m$

distance $L = 200\,km = 2\times10^5\,m$

$$\frac{h}{L} > \frac{1.22\lambda}{D}$$

$$D > 1.46\,m$$

Q6)

$$m\left(\ddot{x} + \gamma\dot{x} + \omega_0^2 x\right) = -eE(t)$$ Lorentz eqn.

for periodic forced $E(t) = E_0 e^{-j\omega t}$

$$\Rightarrow m\frac{d^2x}{dt^2} + m\gamma\frac{dx}{dt} + m\omega_0^2 x = eE_0 e^{-j\omega t}$$

$$x \rightarrow x_0 e^{-j\omega t}$$ obvious. stable soln.

$$m\left(-\omega^2 x_0 + j\gamma\omega x_0 + \omega_0^2 x_0\right) = -eE_0$$

$$x_0 = \frac{-eE_0}{m\left(\omega_0^2 - \omega^2 + j\gamma\omega\right)}$$

$$D = \varepsilon_0 E + P$$

$$\varepsilon_0 \varepsilon E = \varepsilon_0 E + P$$

$$P = \varepsilon_0 E(\varepsilon - 1)$$

$$\varepsilon = \frac{P}{\varepsilon_0 E} + 1$$

$$P = -Nex$$

$$= -Ne\,x_0 e^{-j\omega t}$$

$$= -Ne\,\frac{-eE_0}{m(\omega_0^2 - \omega^2 + j\gamma\omega)}e^{-j\omega t}$$

$$P = \frac{Ne^2}{m(\omega_0^2 - \omega^2 - j\gamma\omega)}E_0 e^{-j\omega t} \rightarrow = E$$

$$\varepsilon = \frac{Ne^2}{\quad} + 1$$

$$\varepsilon = \frac{Ne^2}{m\varepsilon_0 (\omega_0^2 - \omega^2 + j\gamma\omega)} + 1$$

$$(\omega_0^2 - \omega^2) + j(\gamma\omega) \longrightarrow \text{denom}$$

$$\varepsilon = \frac{Ne^2}{m\varepsilon_0 (\omega_0^2 - \omega^2 + j\gamma\omega)} \times \frac{(\omega_0^2 - \omega^2) - j\gamma\omega}{(\omega_0^2 - \omega^2) - j\gamma\omega}$$

$$\text{real} = 1 + \frac{Ne^2 \, (\omega_0^2 - \omega^2)}{\varepsilon_0 m \left((\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2 \right)}$$

$$\text{img} = \frac{Ne^2}{\varepsilon_0 m} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$