PS8

Monday, April 7, 2025 5:10 PM

Q1) a) 12.27

$$\left(E + \frac{h^2}{2m}\frac{d^2}{dx^2}\right)\psi(x) = \sum_{n=-\infty}^{\infty} V_0\,\delta(x-n\Delta)\,\psi(x)$$

$$E\psi(x) = \frac{-h^2}{2m}\frac{d^2\psi(x)}{dx^2} + \sum_{n=-\infty}^{\infty} V_0\,\delta(x-n\Delta)\,\psi(x)$$

integrate from $-\epsilon$ to ϵ & $\epsilon \to 0$

$$\int_0^\epsilon E\psi(x)\,dx = \int_0^\epsilon \frac{-h^2}{2m}\frac{\delta^2\psi}{dx^2}\,dx + \int_0^\epsilon \sum V_0\,\delta(x-n\Delta)\,\psi(x)\,dx.$$

0

$\epsilon \to 0$

$$\frac{-h^2}{2m}\left(\frac{\delta\psi}{dx}\Big|_\epsilon - \frac{\delta\psi}{dx}\Big|_0\right)$$

$V_0\psi(0)$

$$+\frac{h^2}{2m}\left(\psi(0+) - \psi(0-)\right) = V_0\psi(0)$$

$$\psi(x+\Delta) = e^{iq\Delta}\psi(x)$$

$$\psi(x) = Ae^{i(k-k)x} + Be^{i(q+k)x}$$

$$\psi(x) = Ae^{iqx} + Be^{-iqx}$$

$$\frac{h^2}{2m}\frac{1}{dx}\left(Ae^{i(qx-k\Delta)} + Be^{-i(qx+k\Delta)} - Ae^{i(qx)} - Be^{-i(qx)}\right)$$

$$= V_0(A+B)$$

$$\frac{h^2 iq}{2m}\left(A - B - Ae^{i(q-k)\Delta} + Be^{-i(q+k)\Delta}\right) = V_0(A+B)$$

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b) blochs theorem

$$A + B = Ae^{i(q-k)\Delta} + Be^{-i(q+k)\Delta}.$$

$$B = A/(1 - e^{+i(q-k)})$$

$$B = \frac{A\left(1-e^{+i(q-k)}\right)}{\left(e^{-i(q+k)\Delta}-1\right)}$$

$\Rightarrow$ $e^{ix} = \cos x + i \sin x$

imaginary part $\Rightarrow$ $\left(\cos(k\Delta) - \cos(q\Delta) - \frac{mV_0}{q\hbar^2}\sin(q\Delta)\right) A = 0$

$$\Rightarrow \cos(k\Delta) = \cos(q\Delta) + \frac{mV_0\Delta}{\hbar^2}\frac{\sin q\Delta}{q\Delta}$$

$\rightarrow$ 12.29

Q2) $f(E_c) = \dfrac{1}{1+e^{E_g/2kT}}$

$E_g \longrightarrow$ Ge 0.66
Si 1.12
Diamond 5.47

$kT = 8.617\times10^{-5} \times 300$
$= 0.02585\ eV$

Ge $f(E_c) = \dfrac{1}{1+e^{0.66/(2\times0.025)}} \approx 2.88\times10^{-6}$

Si $f(E_c) = \dfrac{1}{1+e^{1.12/2\times0.025}} \approx 4.35\times10^{-10}$

Diamond $f(E_c) = \dfrac{1}{1+e^{5.47/2\times0.025}} \approx 1.1\times10^{-46}$

Q3) 10^{17} As Atoms/cm^3

$n_0 p_0 = n_i^2$ $n_0 = 10^{17}\ cm^{-3}$

$n_0 = 1.5\times10^{10}\ cm^{-3}$

$n_i = 1.5\times10^{10}\ cm^{-3}$

$$p_0 = \frac{n_i^2}{n_0} = 2.25\times10^{3}\ cm^{-3}$$

$$E_f - E_i = kT\ln\left(n_0/n_i\right)$$

$kT = 0.02585\ eV$

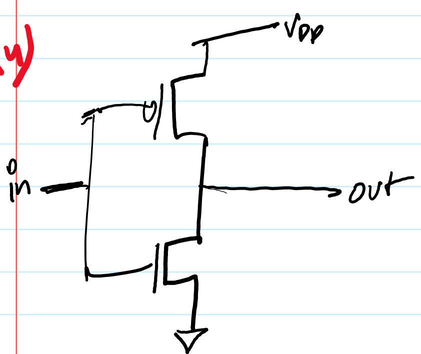
$n_0 = 10^{17}\ cm^{-3}$

$n_i = 1.5\times10^{10}\ cm^{-3}$

$$\ln\left(10^{17}/1.5\times10^{10}\right) \approx 15.7$$

$$E_f - E_i = 0.02585\times15.7 = 0.406\ eV$$

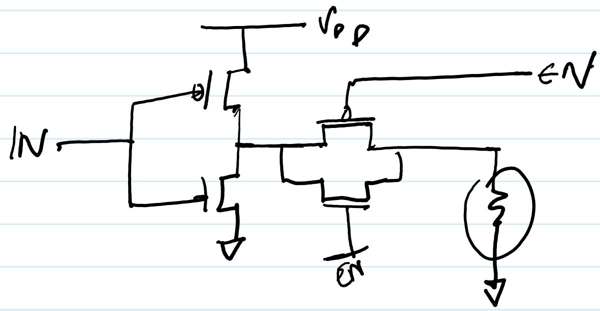
Q4)



CMOS Inverter.

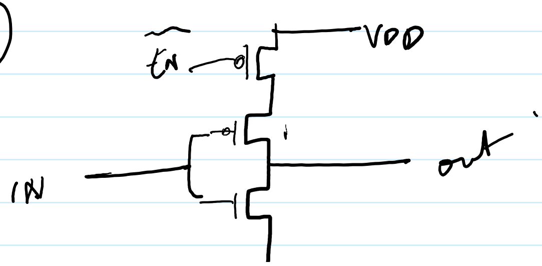
Modified $\overline{EN} = 1\}$ normal inverter op.
$EN = 0\}$ hi-Z state.

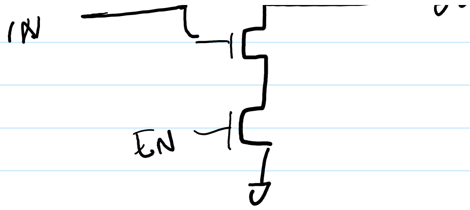
Easiest way to do this is by adding another CMOS pair in series before the output.



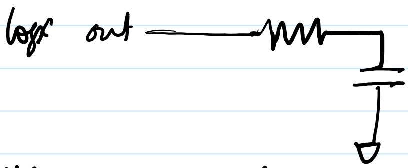
Rload. Adding the final FET (pmos) allows us to control conduction to the load with the EN signal.

(or)





Q5)



a) Energy $= \frac{1}{2} CV^2$ $= \frac{1}{2} 1\times10^{-15} \times 1.8^2$

$= 1.62\times10^{-15} J$

b) Energy lost $= \frac{1}{2} CV^2$

dno

c)

d) $E = CV^2$ net $\Rightarrow$ Power $= f \cdot E_{cycle}$

$1\times10^{-15} \times (1.8)^2 = 3.24\times10^{-15}$

$1 = f \cdot 324\times10^{-15}$

$\Rightarrow f = 3.09\times10^{14}$ Hz

e) $CV^2 = 3.24\times10^{-15}$

$P = N\times f \times E_{cycle} = 10^9 \times 10^9 \times 3.24\times10^{-15}$

$= 3.24W$

f) $Q = CV$

$= 10^{-15} \times 1.8 = 1.8\times10^{-15} C$

$= 10^{-15} \times 1.8 = 1.8 \times 10 \; C$

$e = 1.602 \times 10^{-19} C$

$n = Q/e \approx 11235$