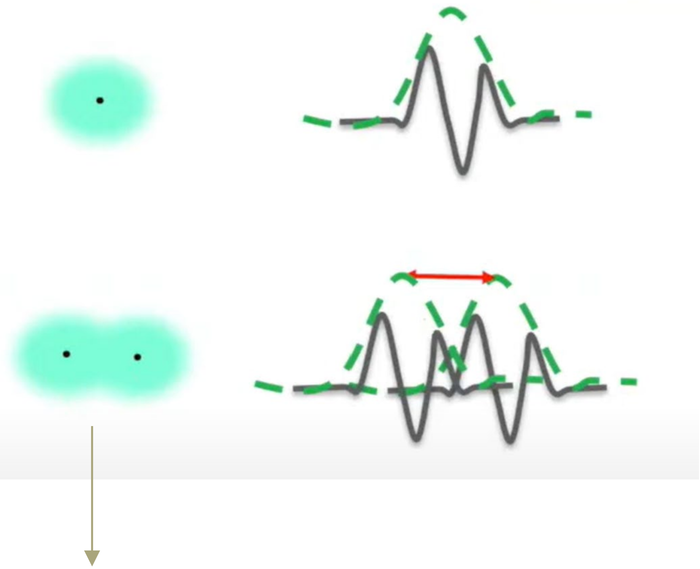
PHYSICS OF INFORMATION TECHNOLOGY 2025
MAS.862

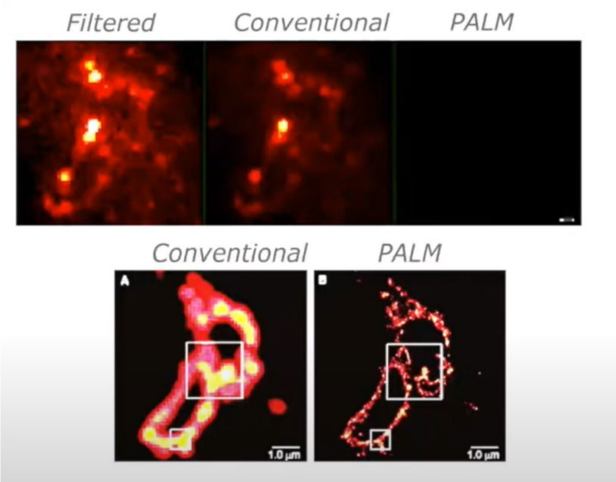
Project Repo: https://gitlab.cba.mit.edu/classes/862.25/site/-/tree/main/people/Shrihari/assignments/ProjectRepo

GETTING TO SUPER-RESOLUTION WITH ULTRASOUND

Shrihari Viswanath

IMAGING LIMITS





Betzig et al., Science, 2006

Rayleigh criterion, minimum distance between two distinguishable objects

Diffraction Limit Calculations for 1.5 MHz Linear Array Transducer

Given Parameters:
c = 1540 — Speed of sound in tissue [m/s]
f = 1.5e6 — Transducer center frequency [Hz]
λ = c / f — Wavelength [m]
n_cycles = 2 — Tone burst cycles (assumed)

Axial Resolution:
SPL = n_cycles * λ — Spatial pulse length [m]
axial_res = SPL / 2 — Axial resolution ≈ λ [m]
Result: ~1.03 mm

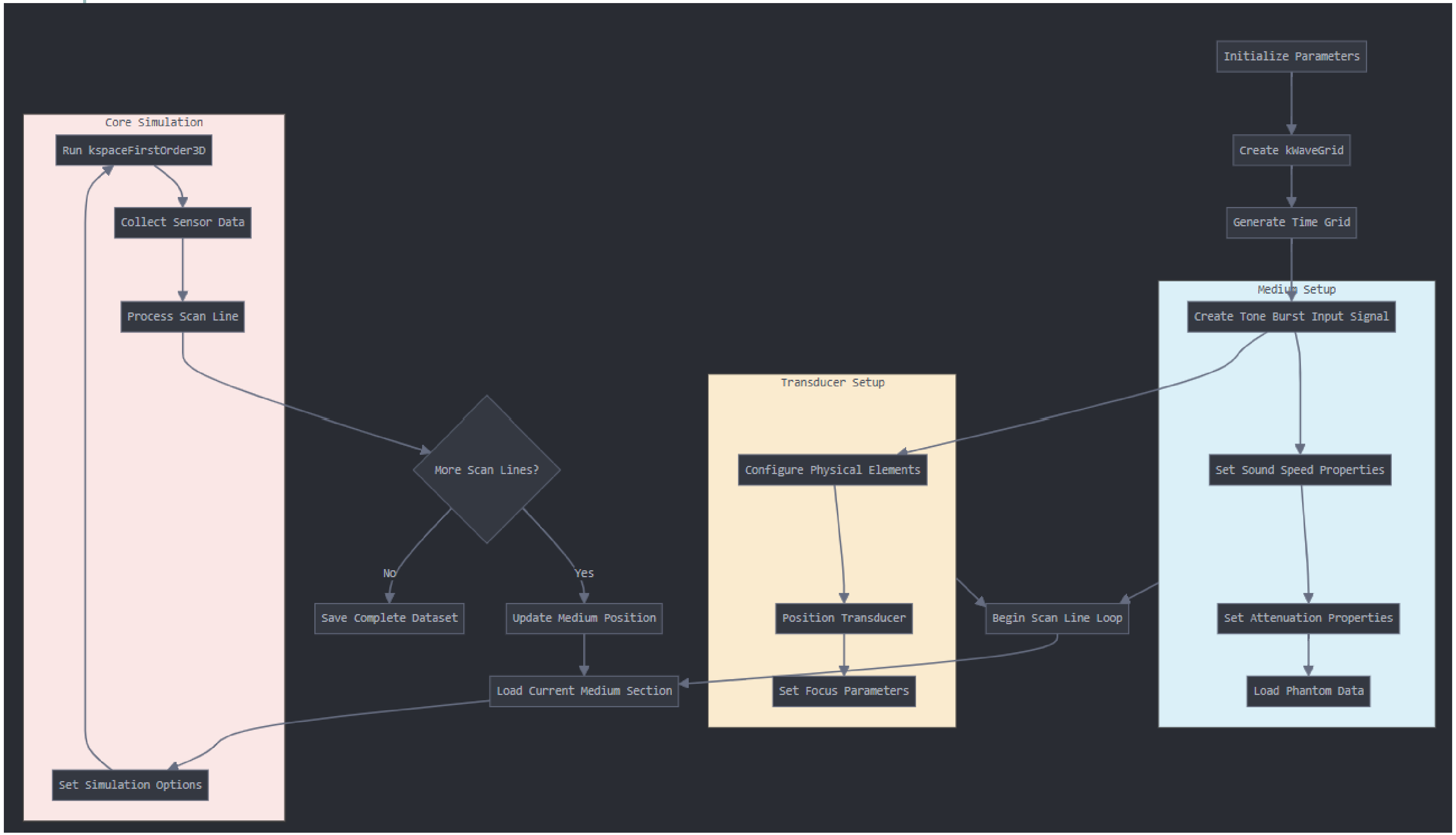
Transducer Parameters:
element_width_mm = 0.078 — Element width
aperture_mm = element_width_mm * 32 — 32 elements
aperture = aperture_mm / 1000 — [m]

Focus Distances
F = 5e-3 — Lateral focus distance [m]
F_elev = 4.75e-3 — Elevation focus distance [m]

Lateral Resolution (Rayleigh criterion)
lat_res = λ * F / aperture — Lateral resolution [m]
Result: ~2.06 mm

Elevation Resolution
elev_res = λ * F_elev / aperture — Elevation resolution [m]
Result: ~1.96 mm

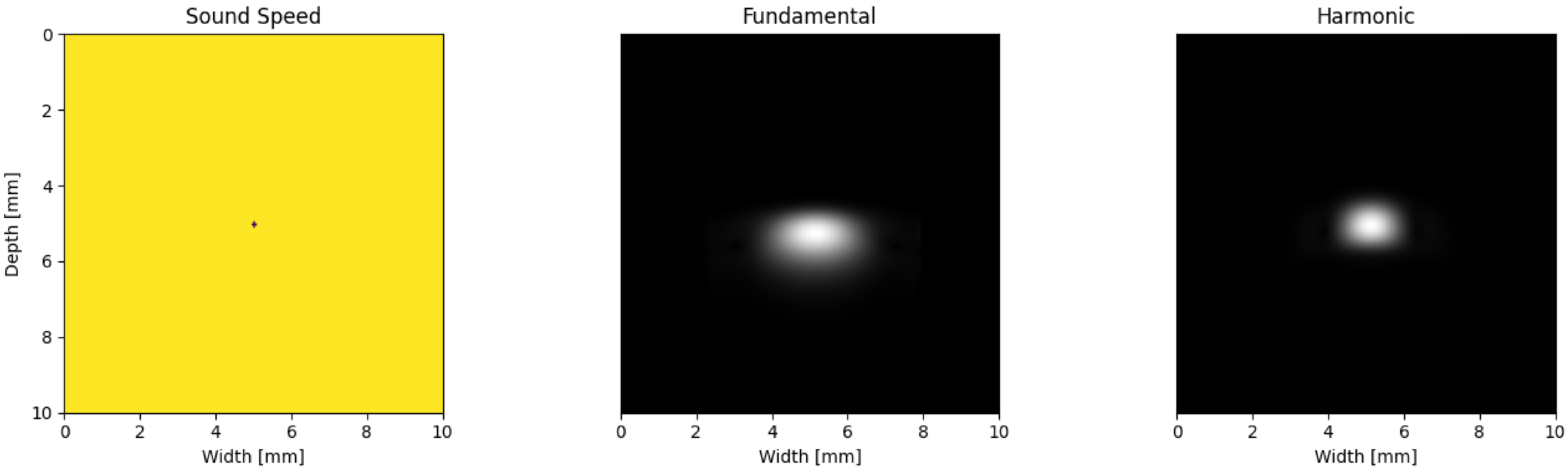
SIMULATOR FLOWCHART

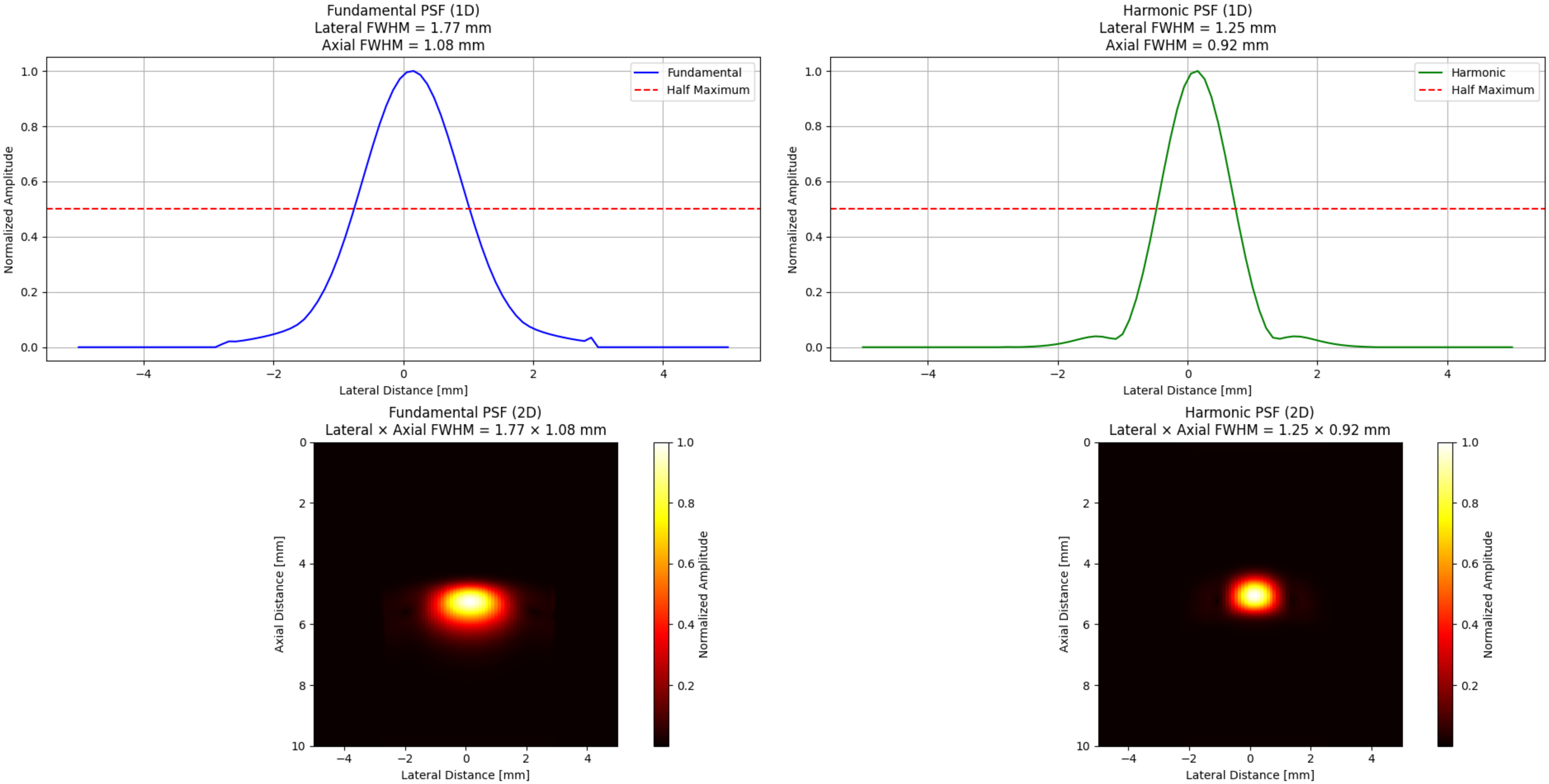


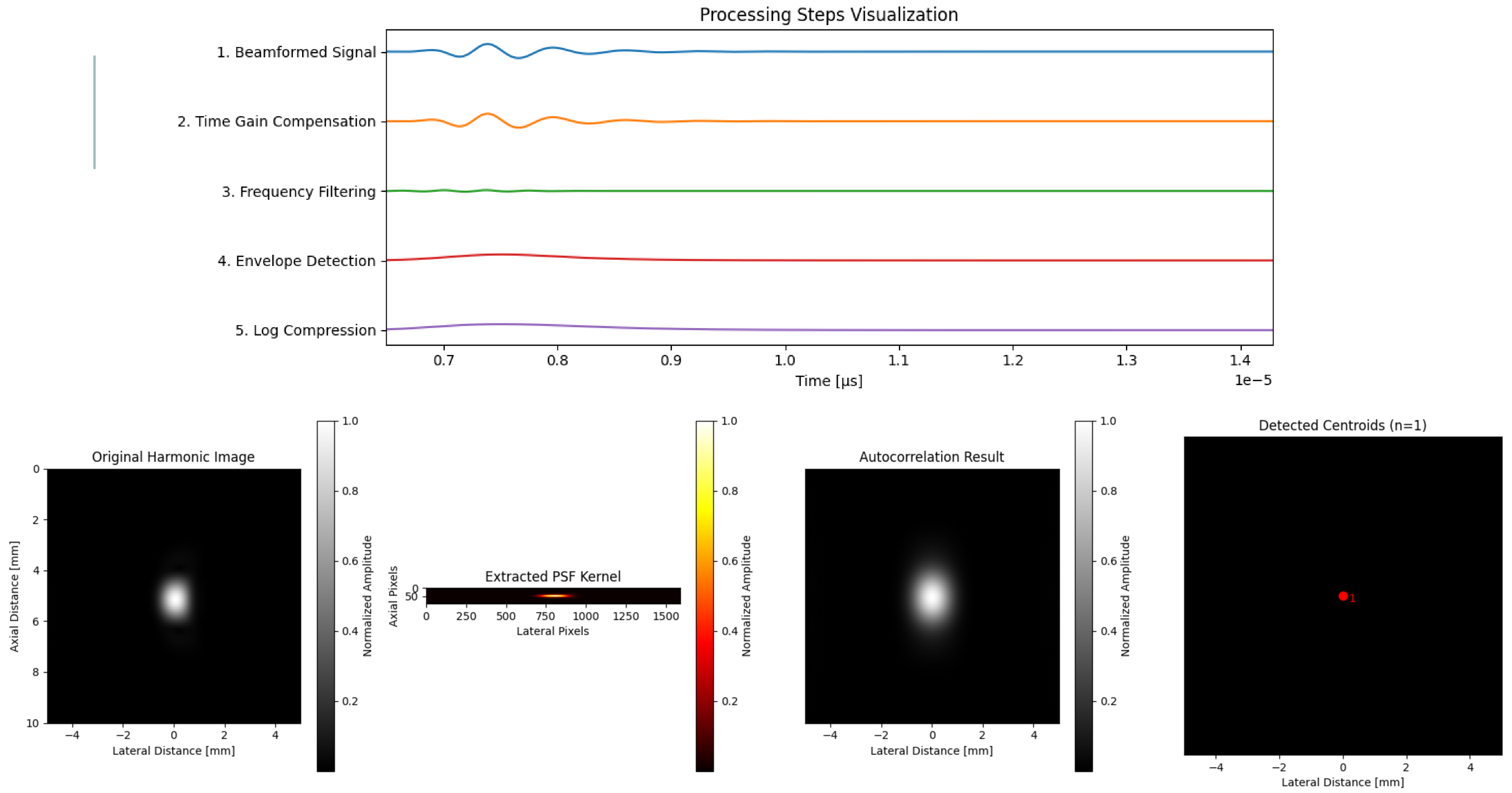
k-Wave uses a k-space pseudospectral method for solving the governing equations, which involves calculating spatial gradients using a Fourier collocation scheme and temporal gradients using a k-space corrected finite-difference scheme.

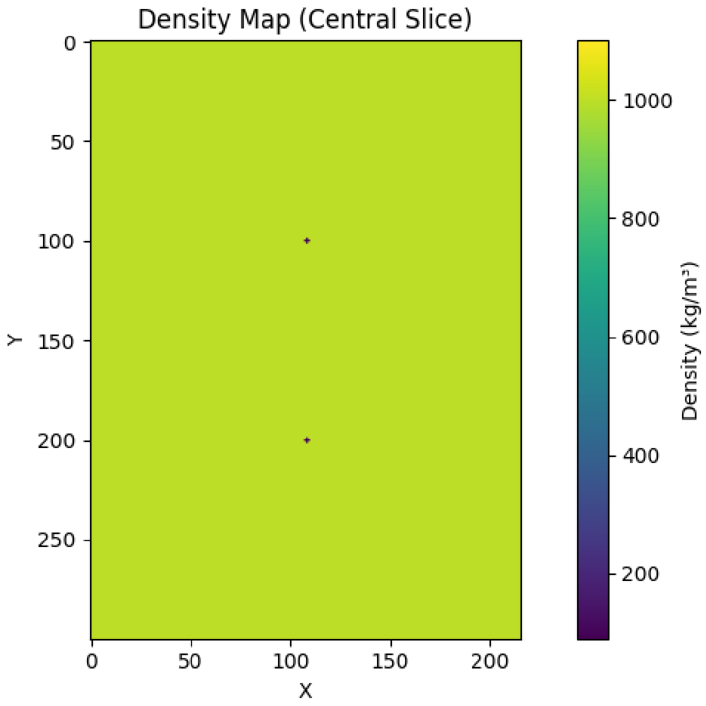
This simulation uses a fine grid size, and the volume has been sliced up and simulated piece by piece.

SIMULATION RESULTS




Fundamental PSF (1D)
Lateral FWHM = 1.77 mm
Axial FWHM = 1.08 mm
Fundamental
Half Maximum
Normalized Amplitude
Lateral Distance [mm]
Harmonic PSF (1D)
Lateral FWHM = 1.25 mm
Axial FWHM = 0.92 mm
Harmonic
Half Maximum
Normalized Amplitude
Lateral Distance [mm]
Fundamental PSF (2D)
Lateral × Axial FWHM = 1.77 × 1.08 mm
Axial Distance [mm]
Lateral Distance [mm]
Normalized Amplitude
Harmonic PSF (2D)
Lateral × Axial FWHM = 1.25 × 0.92 mm
Axial Distance [mm]
Lateral Distance [mm]
Normalized Amplitude


Processing Steps Visualization
1. Beamformed Signal
2. Time Gain Compensation
3. Frequency Filtering
4. Envelope Detection
5. Log Compression
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
Time [µs]
1e−5
Original Harmonic Image
Axial Distance [mm]
Lateral Distance [mm]
Normalized Amplitude
Extracted PSF Kernel
Axial Pixels
Lateral Pixels
Autocorrelation Result
Detected Centroids (n=1)

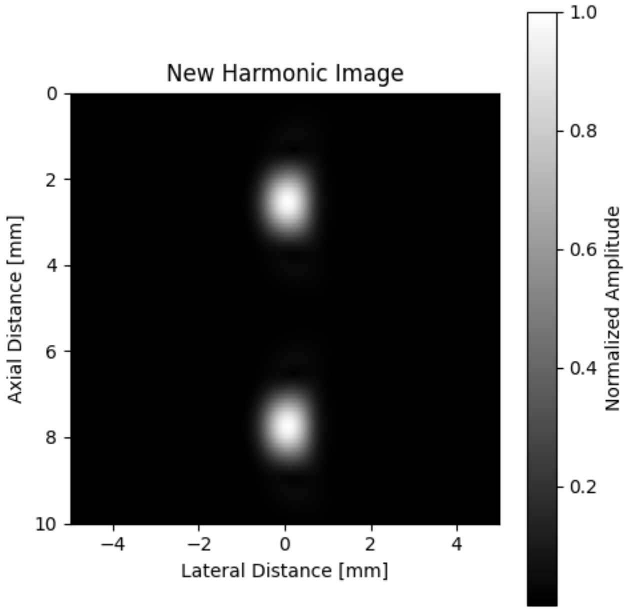


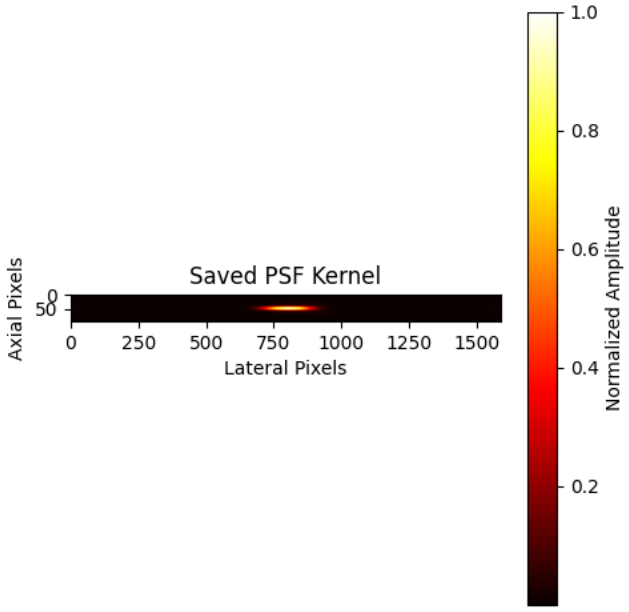
Cramer Rao Lower Bound – theoretical resolution limit for imaging microbubbles in localization microscopy. Dictated by SNR, PSF width and Frame Rate

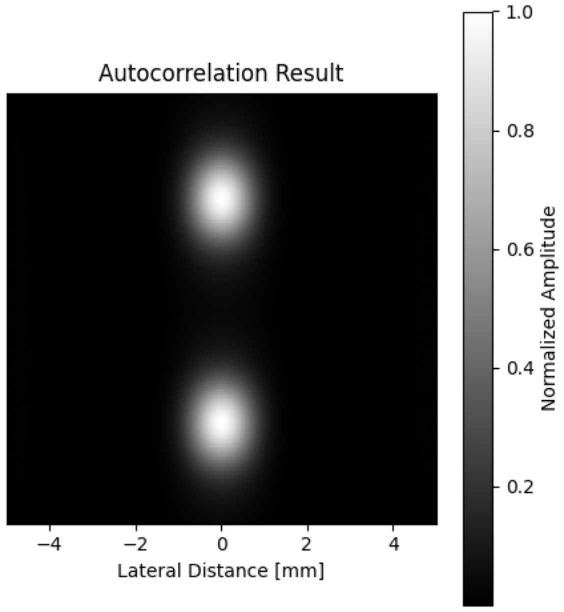
Interesting link between information theory and estimation theory

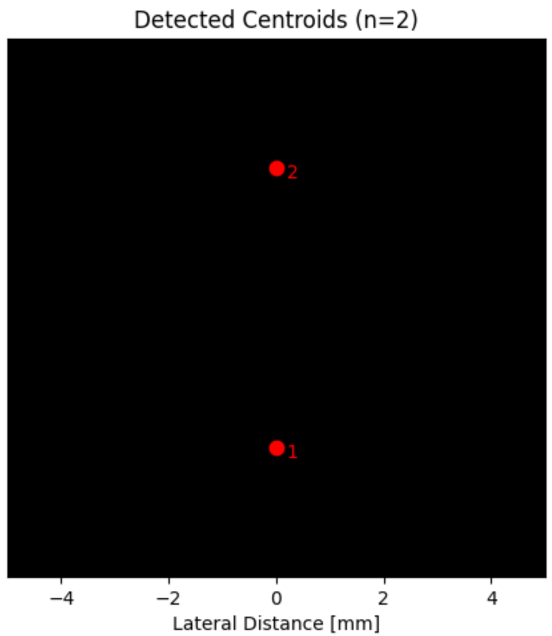
Using high frame rates, low SNR systems researchers have been able to break past the Rayleigh's criterion using bubble tracking and statistical estimations. Allows for more practical bubble densities as a contrast agent.

Ram, Sripad, E. Sally Ward, and Raimund J. Ober. "A stochastic analysis of distance estimation approaches in single molecule microscopy: quantifying the resolution limits of photon-limited imaging systems." Multidimensional systems and signal processing 24 (2013): 503-542.
Rieger, Bernd, and Sjoerd Stallinga. "The Lateral and Axial Localization Uncertainty in Super-Resolution Light Microscopy." ChemPhysChem 15.4 (2014): 664-670.
Stallinga, Sjoerd, and Bernd Rieger. "The effect of background on localization uncertainty in single emitter imaging." 2012 9th IEEE International Symposium on Biomedical Imaging (ISBI). IEEE, 2012.
Chao, Jerry, et al. "A 3D resolution measure for optical microscopy." 2009 IEEE International Symposium on Biomedical Imaging: From Nano to Macro. IEEE, 2009.

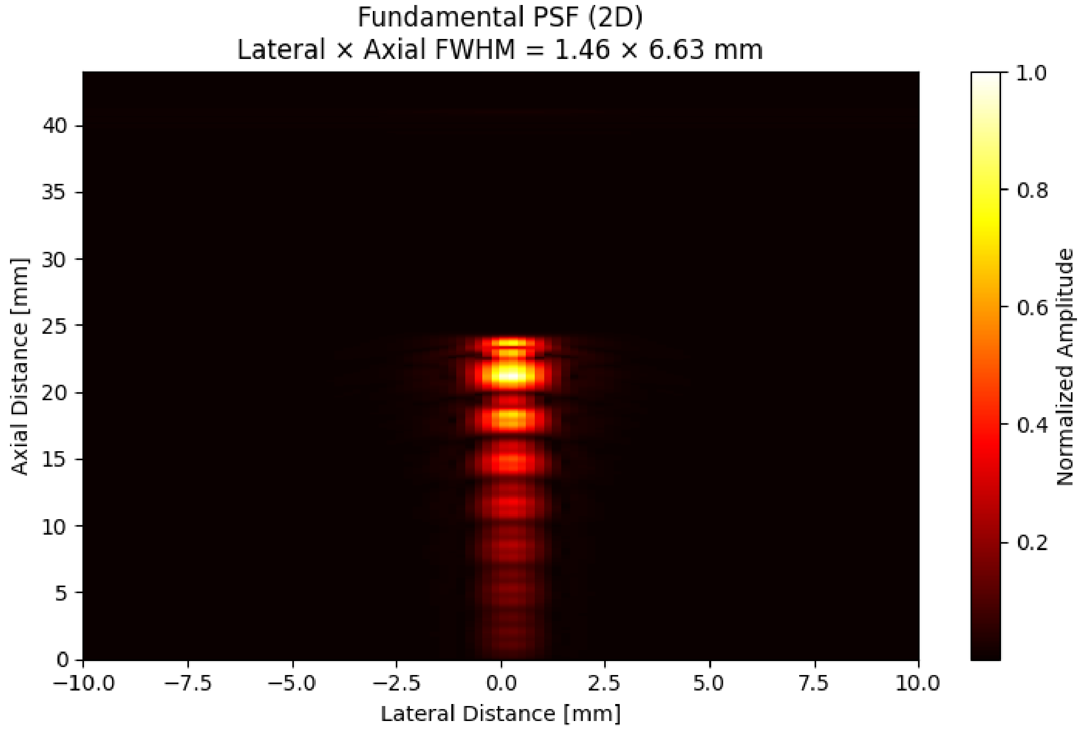


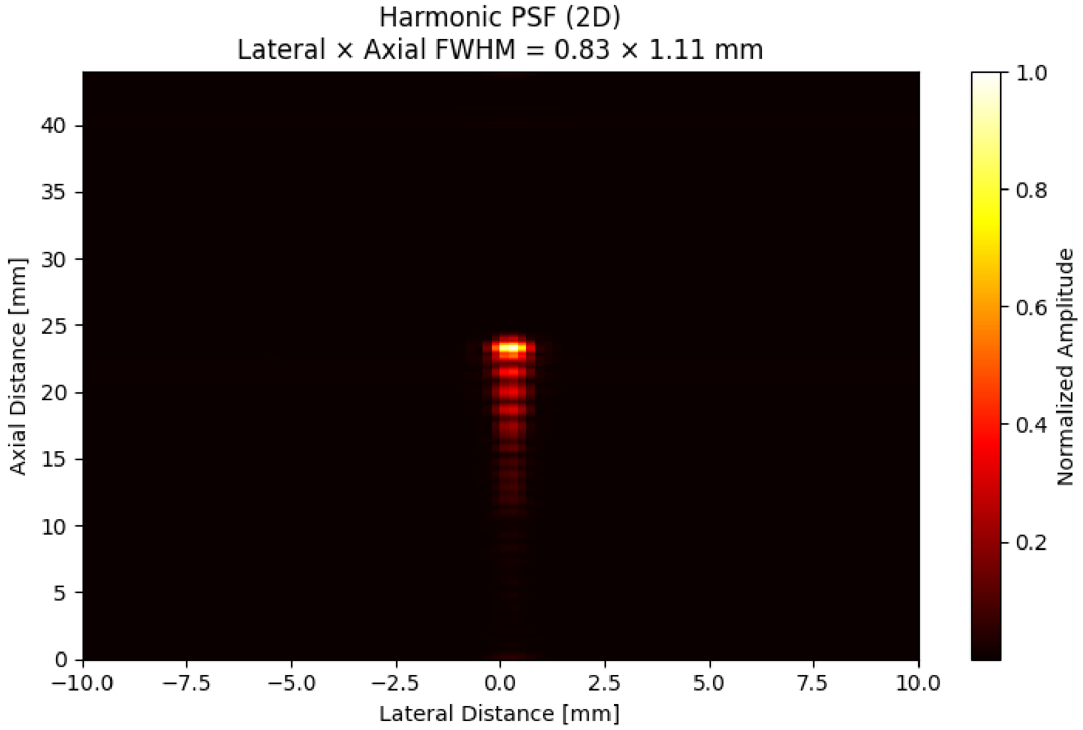






WHY IS A BUBBLE CAUSING THIS BEHAVIOR ?





(11.3) Consider a wave at normal incidence to a dielectric layer with index n_2 between layers with indices n_1 and n_3 (Figure 10.6).

(*a*) What is the reflectivity? Think about matching the boundary conditions, or about the multiple reflections.

(*b*) Can you find values for n_2 and d such that the reflection vanishes?

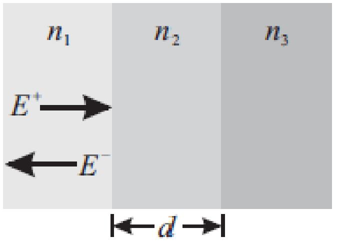
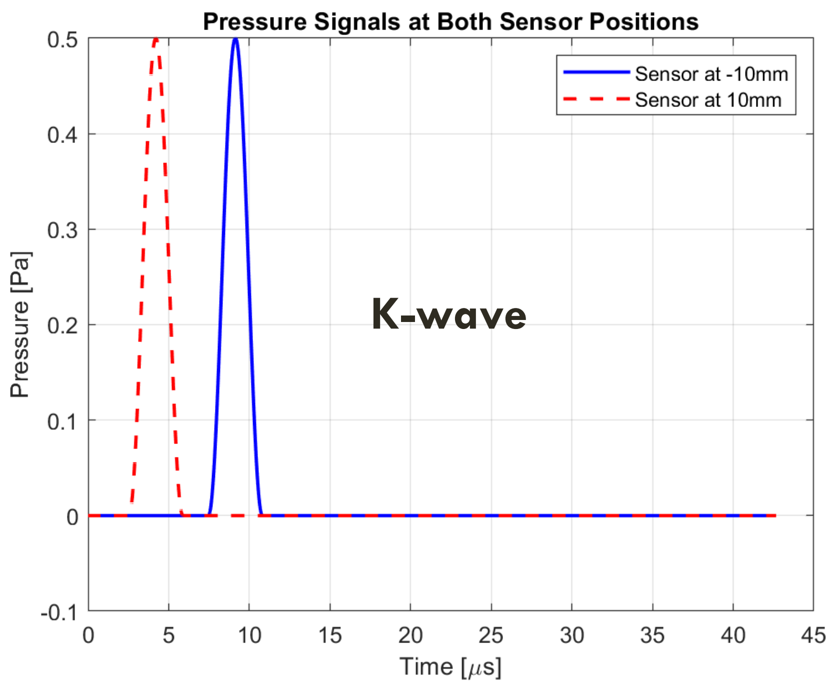
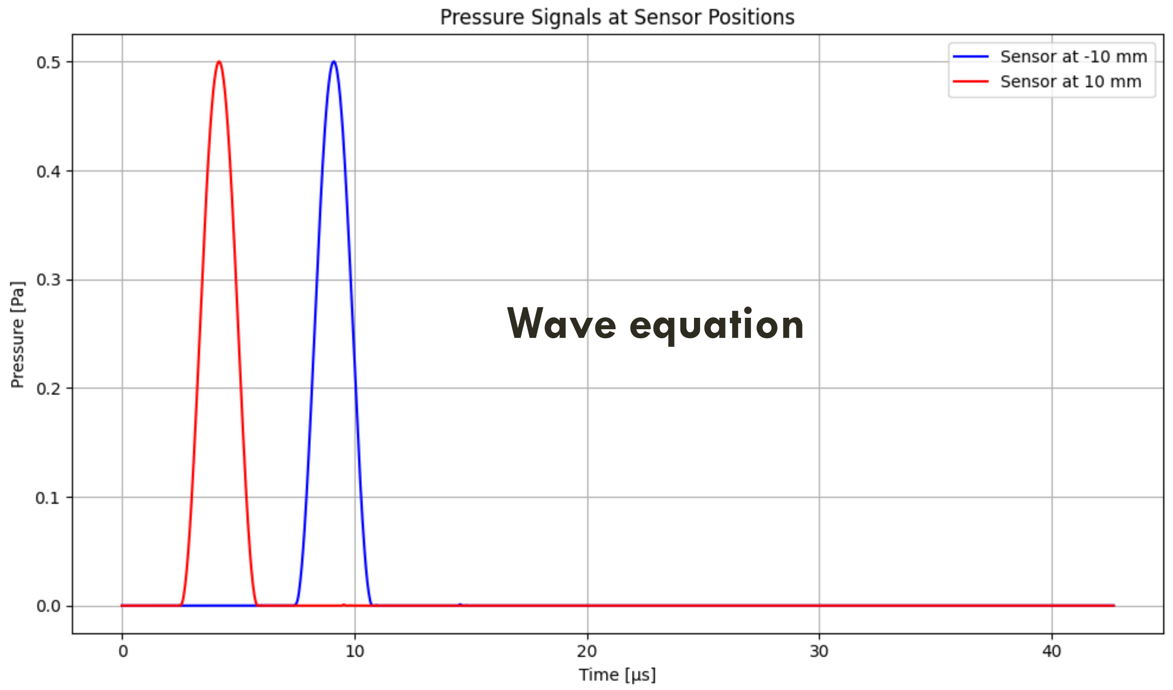


Figure 10.6. Reflection from dielectric interfaces.

VALIDATION





Simulating 1D wave propagation using k-wave and a finite difference method to simulate the wave equation and validate the kwave simulation

VALIDATION

Simulating 2D space with a 1d array and peak pressure was tried to be validated using hand calculation. Doesn't add up perfectly because of the PML introduced in the simulation and due to not being able to exactly calculate the effect of non linearities.

$\rho = 1000$ kg/m^3

$c = 1500$ m/s

$Z = \rho_0 c = $.

Drive $P = 0.2 c^6$

$f_0 = 0.5$ MHz

$\lambda = c/f_0 = 3.08$ mm

$D = 10e^{-3}$ m (Aperture)

$A = D *$ 1mm

$$P_{focus} @ 10mm = \frac{2\rho c V_0 A}{\lambda Z_f} = 2.34 \text{ MPa}$$

Attenuation $\longrightarrow$ 0.75 dB/cm/MHz$^{1.5}$

Source pressure peak ~~peak~~ = 1 MPa ~~→ 0.5MPa peak~~

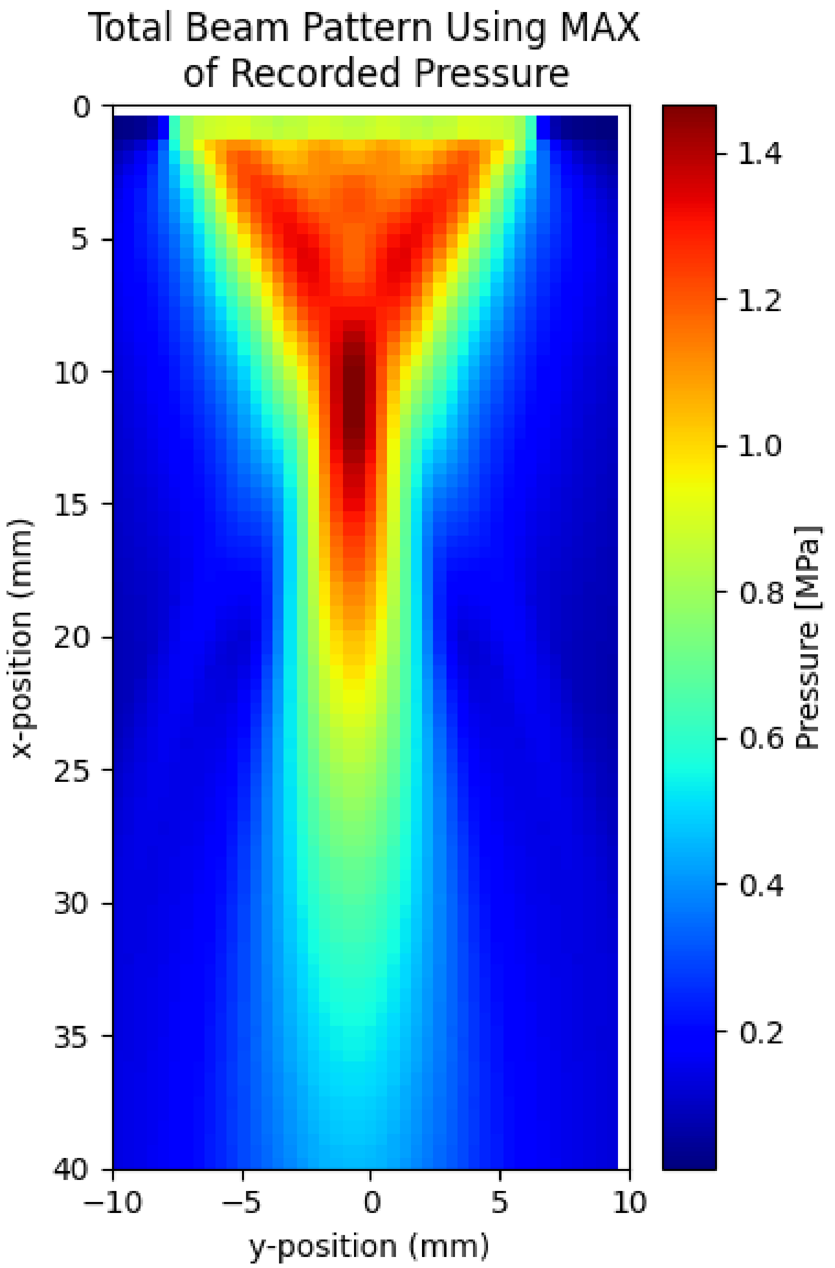
Focal gain ≈ 2.7 Attenuation(X) ≈ 0.97

Non linear gain ≈ 20%

$\Rightarrow 0.5 \times 2.7 \times 0.97 \times 1.2 \approx$ 1.55MPa

further loss due to PML in simulation.

Introducing PML so that we dont deal with excessive ringing and this adds a layer in between that introduces minor losses.



IDEALLY, MEASURE PRESSURE USING A HYDROPOHNE IN THE WATER TANK TO VALIDATE THE SIMULATION.