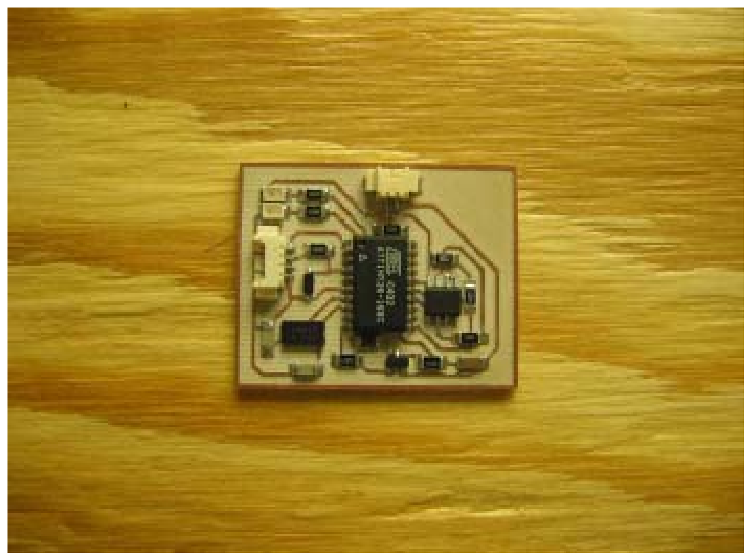
# Internet 0 Demodulation using Mathematical Programming

Andy Sun

NMM Project Presentation



[1]

UDP  Src 18.243.0.1  Dst 18.243.0.1  Port 80

IP header  UDP header  "I0"

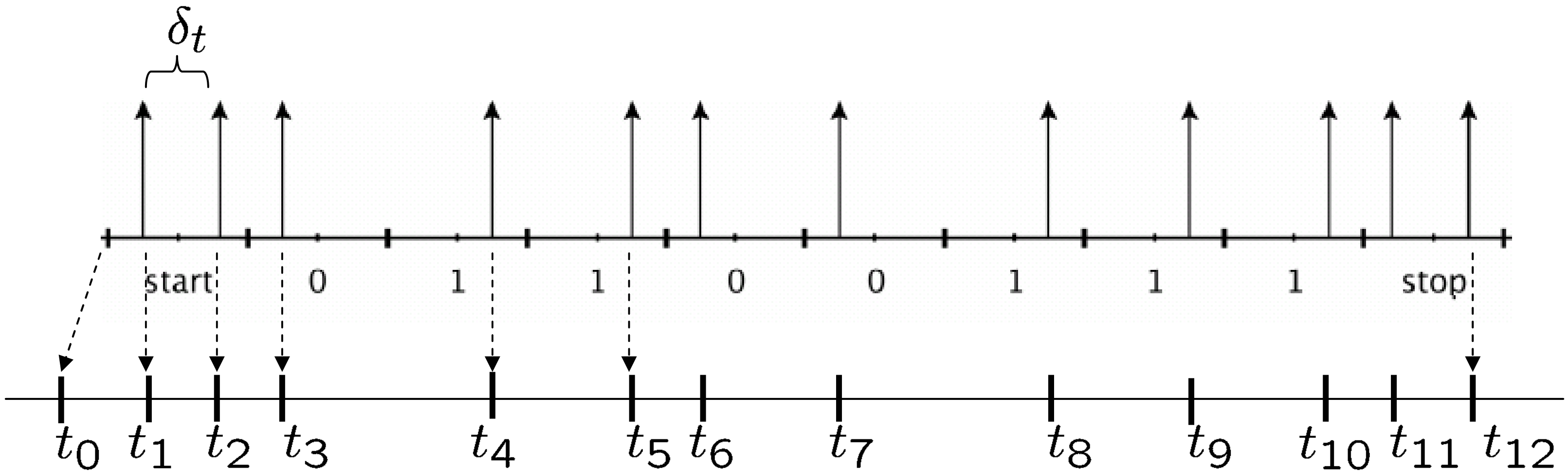
[2]

May 16, 2005

[1]Krikorian, R. Internet 0 Present, conference presentation

[2]Gershenfeld,N.,Krikorian R. and Cohen D. The Internet of Things,Scientific American, 291(Oct. 2004)

# Impulse Sequence with Manchester Encoding




pulse position vector: $t = (t_0, t_1, t_2, ..., t_{12})^T$

bit-sequence vector: $b = (b_1, b_2, ..., b_8)^T$

A nice relation between $\vec{t}$ and $\vec{b}$:

$$t_3 - t_2 = (1 + b_1)\delta_t$$
$$t_4 - t_3 = (2 + b_2 - b_1)\delta_t$$
$$t_5 - t_4 = (2 + b_3 - b_2)\delta_t$$
$$\ldots$$
$$t_{11} - t_{10} = (2 - b_8)\delta_t$$

Define: $\Delta t_1 = (t_3 - t_2)/\delta_t - 1$

$$\Delta t_i = (t_{i+2} - t_{i+1})/\delta_t - 2$$
$$i = 2, ..., 8$$

vector: $\Delta t = (\Delta t_1, ..., \Delta t_8)^T$

A nice relation between $\Delta t$ and $b$:

$$Ab = \Delta t$$

$$A = \begin{bmatrix} 1 & & & & & & & \\ -1 & 1 & & & & & & \\ & -1 & 1 & & & & & \\ & & -1 & 1 & & & & \\ & & & -1 & 1 & & & \\ & & & & -1 & 1 & & \\ & & & & & -1 & 1 & \\ & & & & & & -1 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & & & & & & & \\ 1 & 1 & & & & & & \\ 1 & 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & & & & \\ 1 & 1 & 1 & 1 & 1 & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

therefore, $b_1$ is $\Delta t_1$

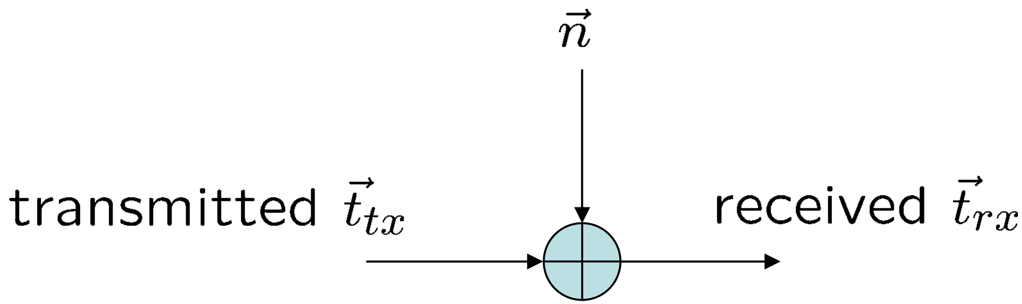
$b_2$ is the sum $\Delta t_1 + \Delta t_2$

$b_3$ is the sum $\Delta t_1 + \Delta t_2 + \Delta t_3$

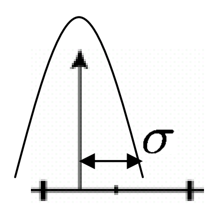
...

In another word, $b$ is the first eight elements of the convolution sum of $\mathbf{1}$ and $\Delta t$

Channel Model:




Pulse position perturbed by Gaussian noise $\mathcal{N}(0, \sigma^2)$



## Demodulation by Mathematical Programming

$$\min_{t_0,\delta_t} \sum_{i=1}^{12} \min(d_{i1}(t_0,\delta_t,t_i), d_{i2}(t_0,\delta_t,t_i)) \qquad (1)$$

where $d_{i1}(t_0,\delta_t,t_i) = |t_0 + (2i+0.5)\delta_t - t_i|$

$$d_{i2}(t_0,\delta_t,t_i) = |t_0 + (2i+1.5)\delta_t - t_i|, i = 2, .., 10$$

$$d_{11} = d_{12} = |t_0 + \delta_t - t_1|$$

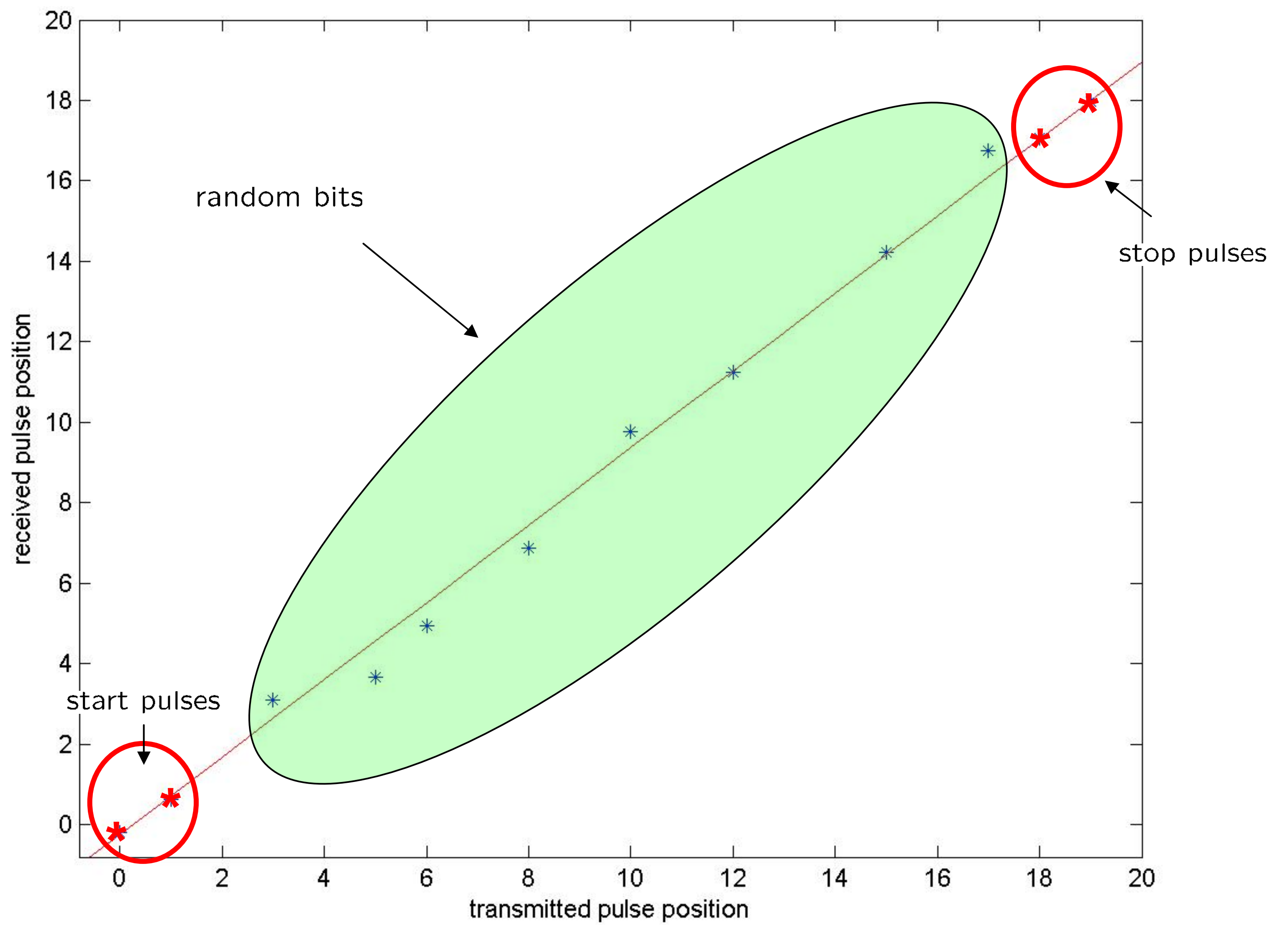
$$d_{21} = d_{22} = |t_0 + 1.5\delta_t - t_1|$$

$$d_{111} = d_{112} = |t_0 + 18.5\delta_t - t_1|$$

$$d_{121} = d_{122} = |t_0 + 19.5\delta_t - t_1|$$

Simple implementation:linear regression on $t_0, \delta_t$

Observation:



Simple regression model:

$$y = t_0 + \delta_t \cdot k$$

where $k = [0.5, 1.5, 18.5, 19.5]^T$

$y = [t_1, t_2, t_{11}, t_{12}]^T$, i.e. start/stop pulses

Estimate pulse positions $\tilde{t} = [\tilde{t}_1, ..., \tilde{t}_8]^T$ from $t_0, \delta_t$:

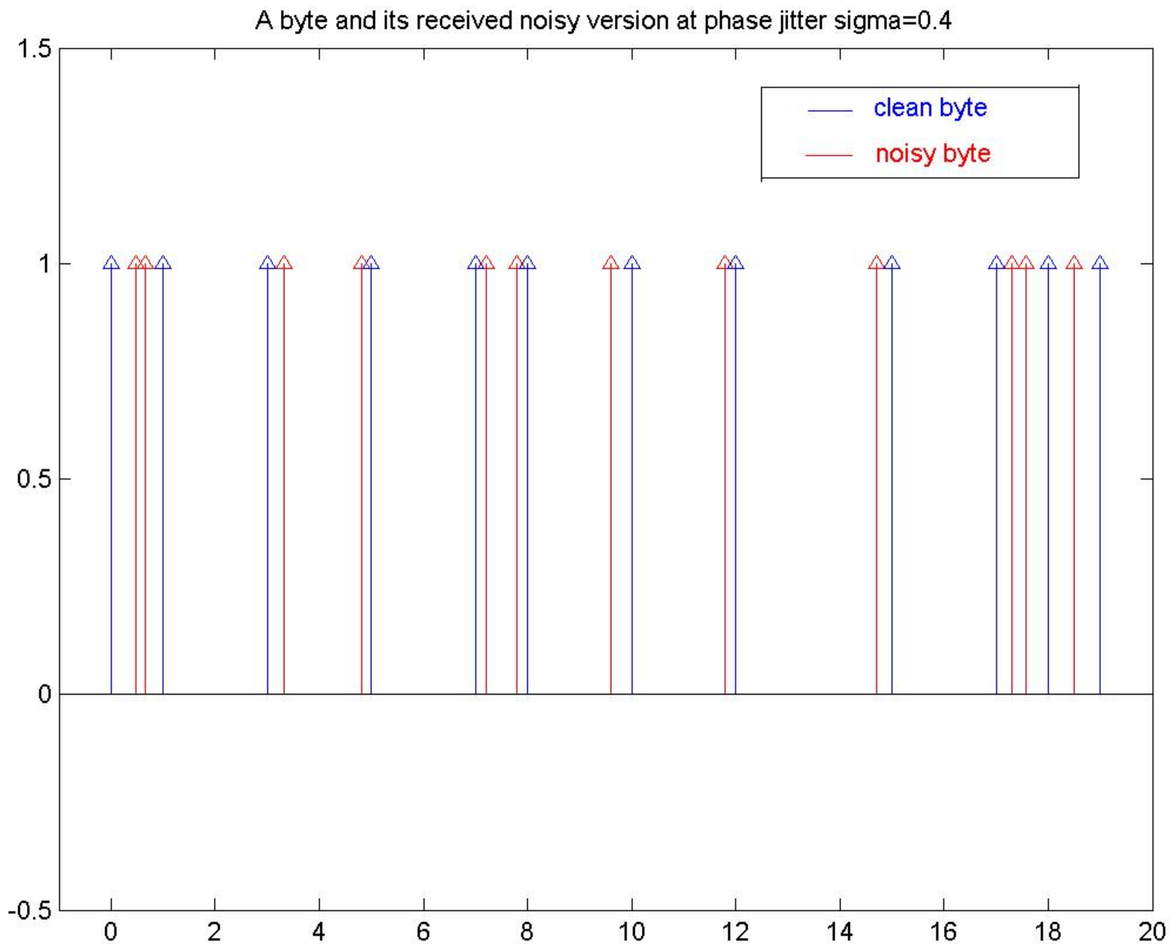
$$\min |\tilde{t}_i - t_i|$$

$$\text{s.t.} \tilde{t}_i = \{f_{i1}(t_0, \delta_t), f_{i2}(t_0, \delta_t)\}$$

where:

$$f_{i1}(t_0, \delta_t) = t_0 + (2i + 0.5)\delta_t$$

$$f_{i2}(t_0, \delta_t) = t_0 + (2i + 1.5)\delta_t$$

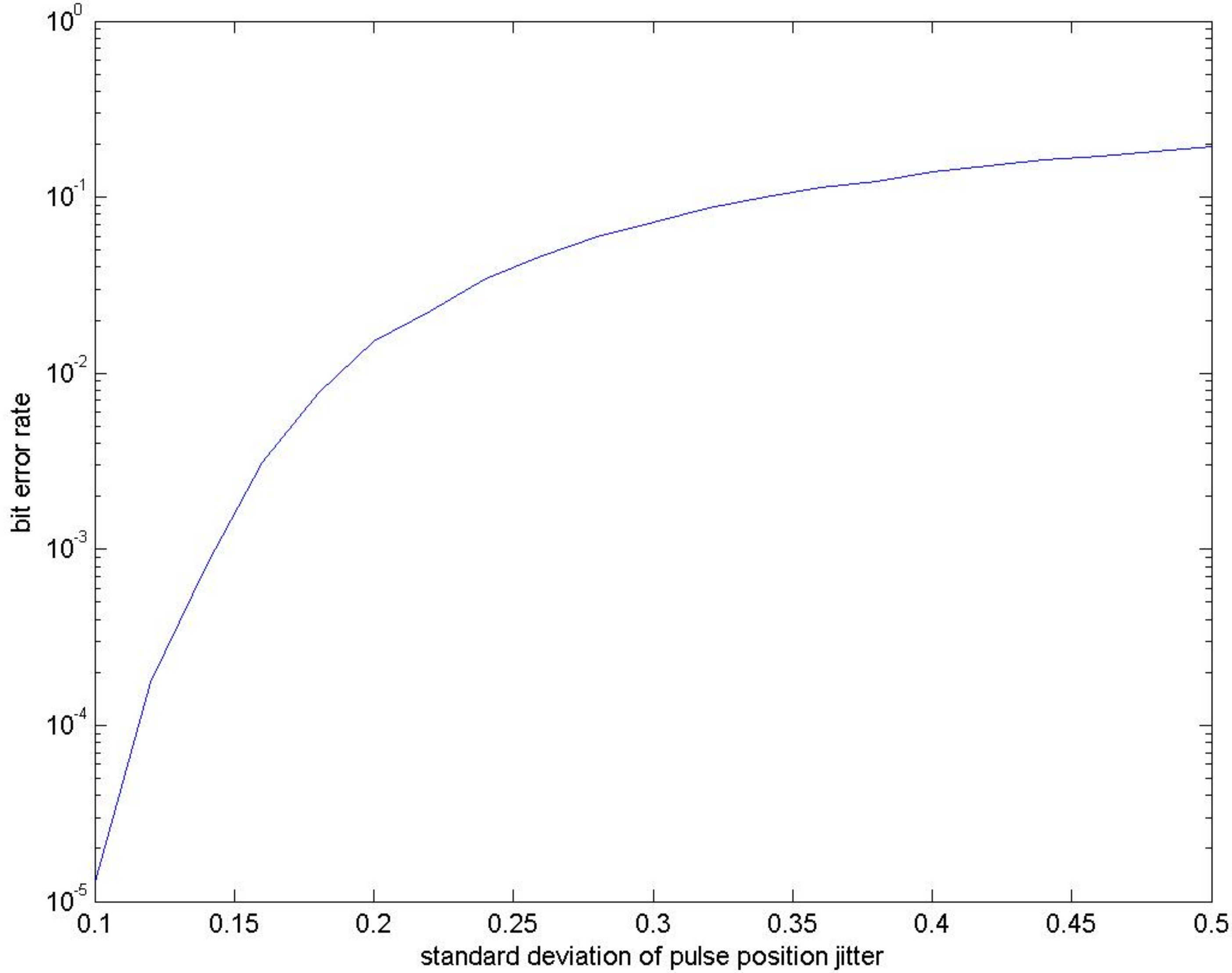



Clean byte [00011100] has $t_0 = -0.5, \delta_t = 1$

Estimated $t_0 = -0.3925, \delta_t = 0.9695$

and the byte is correctly demodulated

# Simulation bit-error-rate v.s. phase noise:



## Demodulation by Mathematical Programming

$$\min_{t_0,\delta_t} \sum_{i=1}^{12} \min(d_{i1}(t_0,\delta_t,t_i), d_{i2}(t_0,\delta_t,t_i)) \qquad (1)$$

where $d_{i1}(t_0,\delta_t,t_i) = |t_0 + (2i+0.5)\delta_t - t_i|$

$$d_{i2}(t_0,\delta_t,t_i) = |t_0 + (2i+1.5)\delta_t - t_i|, i = 2,..,10$$

$$d_{11} = d_{12} = |t_0 + \delta_t - t_1|$$

$$d_{21} = d_{22} = |t_0 + 1.5\delta_t - t_1|$$

$$d_{111} = d_{112} = |t_0 + 18.5\delta_t - t_1|$$

$$d_{121} = d_{122} = |t_0 + 19.5\delta_t - t_1|$$

(1)$\Rightarrow$

$$\min_{t_0,\delta_t} \sum_{i=1}^{12} \max(-d_{i1}, -d_{i2})$$

$$\Longrightarrow \quad \begin{array}{l} \min\limits_{t_0,\delta_t,z} \sum\limits_{i=1}^{12} z_i \\ \text{s.t. } z_i \geq -d_{i1}, \\ z_i \geq -d_{i2} \\ i = 1, 2, ..., 12 \end{array} \quad \Longrightarrow \quad \begin{array}{l} \min\limits_{t_0,\delta_t,z} \mathbf{1}^T z \\ \text{s.t. } z_i \geq -d_{i1}, \\ z_i \geq -d_{i2} \\ i = 1, 2, ..., 12 \end{array}$$

linear cost, absolute-value(quadratic) constraints

Questions?

simulation of the optimization problem in (1)