Dhaval Adjodah
Week 5

10.1

a) $$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2}$$

$$\int_u^v \varphi_j \frac{d^2\varphi_i}{dx^2}\, dx = \varphi_j \frac{d\varphi_i}{dx} - \int \frac{d\varphi_i}{dx} \frac{d\varphi_j}{dx}\, dx.$$

Using Galerkin expansion, for each basis function φ_j,

$$\int \left(\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial}{\partial x} \frac{\partial^2 u}{\partial x^2} \right) \varphi_j \, dx = 0$$

$$\frac{d}{dt}\left(a_i(t)\, \varphi_j \frac{d^2\varphi_i}{dx^2} \right)$$

For $u(x,t) = \sum_i a_i(t)\, \varphi_i(x)$,

$$\Rightarrow \sum_i \int \left\{ \frac{d^2 a_i}{dt^2} \varphi_i \varphi_j - v^2 a_i \varphi_j \frac{d^2\varphi_i}{dx^2} - \gamma \varphi_j \frac{da_i}{dt} \frac{d^2\varphi_i}{dx^2} \right) dx = 0$$

$$\Rightarrow \sum_i \frac{d^2 a_i}{dt^2} \underbrace{\int_0^1 \varphi_i \varphi_j \, dx}_{A_{ij}} - \sum_i \left(v^2 a_i + \gamma \frac{da_i}{dt} \right) \underbrace{\left\{ -\int_0^1 \frac{d\varphi_i}{dx} \frac{d\varphi_j}{dx}\, dx + \varphi_j \frac{d\varphi_i}{dx} \Big|_0^1 \right\}}_{B_{ij}} = 0$$

for interval (0,1)

Hence, $$A \cdot \frac{d^2\vec{a}}{dt^2} - B \cdot \left(v^2 \vec{a} + \gamma \frac{d\vec{a}}{dt} \right) = 0$$

b) Generic hat function is

$$\varphi_i = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}} & = \frac{x - (x_i)}{h} \\ \frac{x_{i+1} - x}{x_{i+1} - x_i} & = \frac{x_{i+1} - x}{h} \\ 0 & = 0 \end{cases}$$