19.1 Kalman gain matrix in limit of small measurement noise

$N^y_t \longrightarrow 0$

$$K = E_{t|t-1} B_t^T \left( B_t E_{t|t-1} B_t^T + N^y_t \right)^{-1}$$

$\frac{1}{a+\varepsilon}$ and $\varepsilon \to 0$

series w/ lowest order term

$$\frac{1}{a\left(1+\frac{\varepsilon}{a}\right)} = \frac{1}{a} \frac{1}{1+\varepsilon/a} = \frac{1}{a}\left(1-\varepsilon/a\right)$$

$$K = E_{t|t-1} B_t^T \left( B_t E_{t|t-1} B_t^T \left( I + (B_t E_{t|t-1} B_t^T)^{-1} N^y_t \right) \right)^{-1}$$

$$B_t^{-1} \left( I - N^y_t \left( B_t E_{t|t-1} B_t^T \right)^{-1} \right)$$

error matrices

error update is no longer recursive

$$E_{t+1|t} = A_t B_t^{-1} N^y_t B_t^{T\,-1} A_t^T + N^x_t$$

no need to track history.

(19.2) periodically modulated sin w/ noise:

$$y_n = \sin(0.1 t_n + 4\sin(0.01\, t_n)] + \eta$$

$$\equiv \sin(\theta_n) + \eta$$

$\eta$ = Gauss noise process

$\sigma = 0.1$

two component state vector

$$x_n = (\theta_n, \theta_{n-1})$$

assume linear extrapolation:

$$\theta_{n+1} = \theta_n + (\theta_n - \theta_{n-1}) = 2\theta_n - \theta_{n-1}$$

$N^x$ is diagonal (system noise matrix)

$$E^x_{t|t-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ initial error estimation

$$x_{t+1|t} = A_t \cdot \vec{x}_{t|t} \rightarrow A_t = \begin{bmatrix} [2, & -1] \\ [1, & 0] \end{bmatrix}$$

$$\vec{y}_{t|t-1} = B_t \cdot x_{t|t-1}$$

$$B_t = \cos(\theta_n)(\theta_n - \theta_{n-1})$$

$$= [\cos(\theta_n), -\cos(\theta_n)]$$

$$N^x = \begin{bmatrix} noise^2 & 0 \\ 0 & noise^2 \end{bmatrix}$$