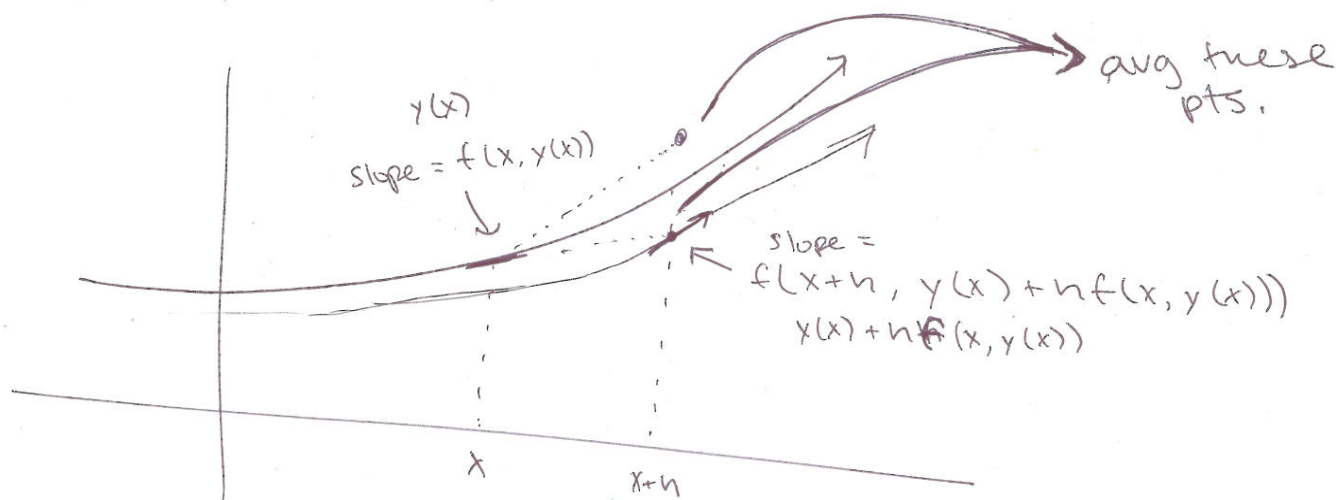


7.1 Heun method. What is second order approx error?

$$y(x+h) = y(x) + \frac{h}{2}$$

$$\rightarrow \left[ f(x, y(x)) + f(x+h, y(x) + hf(x, y(x))) \right]$$



second order Taylor expansion:

$$y(x+h) = y(x) + hf(x, y(x)) + \frac{h^2}{2} \left[ \frac{df}{dx} + f \frac{df}{dy} \right] + O(h^3)$$

$$= y(x) + hf(x, y(x)) + \frac{h^2}{2} \frac{d}{dx} f(x, y(x)) + O(h^3)$$

$$f(x+h, y(x) + hf(x, y(x))) + O(h^3)$$

$$\hookrightarrow f(x, y(x)) + h \frac{d}{dh} f(x+h, y(x) + hf(x, y(x))) + O(h^2)$$

$$O(h^2) - \frac{h}{2} = O(h^3)$$