Problem 3.1, e.)

Find the Solution to the ode below by using Laplace transforms. Given the initial condition $x(0) = \dot{x}(0) = 0$

$$m\ddot{x} + \gamma \dot{x} + kx = e^{i\omega t}$$

$$\mathcal{L}\{m\ddot{x} + \gamma \dot{x} + kx = e^{i\omega t}\}$$

$$m\{s^2 F(s) - s(x(0)) - \dot{x}(0)\} + \gamma\{s F(s) - x(0)\} + k F(s) = \frac{1}{(s - i\omega)}$$

Sub in $x(0) = 0$, $\dot{x}(0) = 0$

$$m\ s^2 F(s) + \gamma s F(s) + k F_s = \frac{1}{s - i\omega}$$

$$F(s)\left(s^2 + \frac{\gamma}{m} s + \frac{k}{m}\right) = \frac{m}{s - i\omega}$$

$$F(s) = \frac{m}{\left(s^2 + \frac{\gamma_s}{m} + \frac{k}{m}\right)(s - i\omega)}$$

$$S_{1,2} = \frac{-\frac{\gamma}{m} \pm \left[\left(\frac{\gamma}{m}\right)^2 - 4\left(\frac{k}{m}\right)\right]^{1/2}}{2} = \frac{-\gamma}{2m} \pm \left[\left(\frac{\gamma}{2m}\right)^2 - \frac{k}{m}\right]^{1/2}$$

Sub in $\omega^2 = \frac{k}{m}$

$$S_1 = \frac{-\gamma}{2m} + \left[\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right]^{1/2} \qquad S_2 = \frac{-\gamma}{2m} - \left[\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right]^{1/2}$$

Continuing 3.1, e)

$$F(s) = m\left[s + \frac{\gamma}{2m} - \left[\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right]^{1/2}\right]^{-1}\left[s + \frac{\gamma}{2m} + \left[\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right]^{1/2}\right]^{-1}(s - i\omega)^{-1}$$

$$a = \frac{\gamma}{2m} - \left(\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right)^{1/2}$$

$$b = \frac{\gamma}{2m} + \left[\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right]^{1/2}$$

$$c = -i\omega$$

$$F(s) = \frac{m}{(s+a)(s+b)(s+c)}$$

$$\mathcal{L}^{-1}[F(s)] = \left[\frac{e^{-at}}{(a-b)(a-c)} - \frac{e^{-bt}}{(a-b)(b-c)} - \frac{e^{-ct}}{(a-c)(c-b)}\right]m$$

$$\boxed{x(t) = \frac{e^{-at}}{(a-b)(a-c)} - \frac{e^{-bt}}{(a-b)(b-c)} - \frac{e^{-ct}}{(a-c)(c-b)}}$$

where:

$$a = \frac{\gamma}{2m} - \left(\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right)^{1/2}$$

$$b = \frac{\gamma}{2m} + \left(\left(\frac{\gamma}{2m}\right)^2 - \omega^2\right)^{1/2}$$

$$c = -i\omega$$