Problem 3.3

A common simple digital filter used for smoothing a signal is

$$y(k) = \alpha y(k-1) + (1-\alpha)x(k)$$

α - a parameter that determines the response of the filter

Use z-transformations to solve for $y(k)$ as a function of $x(k)$ (assume $y(k<0) = 0$. What is the amplitude of the frequency response.

$$Z\left\{ y(k) = \alpha y(k-1) + (1-\alpha)x(k) \right\}$$

$$Y(z) = \alpha\left(Y(z)z^{-1} + y(-1)\right) + (1-\alpha)X(z)$$

$$y(-1) \Rightarrow 0$$

$$Y(z)\left(1 - \frac{\alpha}{z}\right) = (1-\alpha)X(z)$$

$$Y(z) = \frac{(1-\alpha)}{\left(1-\frac{\alpha}{z}\right)}X(z) = \frac{z(1-\alpha)}{z-\alpha}X(z)$$

$$z^{-1}\left[Y(z)\right] = \alpha^{k}(1-\alpha)x(k)$$

$$\boxed{y(k) = \alpha^{k}(1-\alpha)\,x(k)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z(1-\alpha)}{z-\alpha}$$

Problem 3.3 continued

assuming the input is $x(k) = e^{i\omega\delta_t k}$

$$y(k) = \lim_{k \to \infty} \sum_{n=0}^{k} h(n)\, x(k-n) \quad {}^{*}\text{where } n=0$$

$$y(k) = \lim_{k \to \infty} \sum_{n=0}^{k} h(n)\, e^{i\omega\delta_t (k-)}$$

$$y(k) = H(e^{i\omega\delta_t})$$

frequency response

$$\boxed{y(k) = \frac{e^{i\omega\delta_t}(1-\alpha)}{e^{i\omega\delta_t} - \alpha}}$$