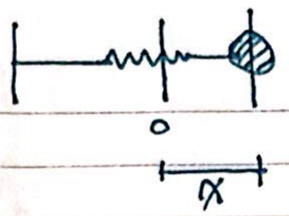
3.8 Pset



a) If there is no external force, by Newton's second law.

$F = m\ddot{x} = -kx - c\dot{x}$

$0 = m\ddot{x} + c\dot{x} + kx$

$0 = \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x$

if this is undamped, $F_{friction} = 0$, So $\gamma = 0$.

If there is external force the force should be $-\gamma\dot{x}$

$\therefore \; -\gamma\dot{x} + e^{i\omega t} = 0$

b) homogeneous, $m\ddot{x} + \gamma\dot{x} + kx = 0$

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \frac{k}{m}x = 0 \quad \cdots ①$$

let's $z = Ae^{i(pt+\alpha)}$

① is $\quad z\left(-p^2 + \frac{\gamma}{m}\cdot ip + \frac{k}{m}\right) = 0$

Since z, ocillation, cannot be 0

$$-p^2 + \frac{\gamma}{m}\cdot ip + \frac{k}{m} = 0 \quad \begin{cases} \text{real} \to 0 \\ \text{imaginary} \to 0 \end{cases} \quad \cdots ②$$

let's $p = n + i\cdot s$

$p^2 = n^2 - s^2 + 2nsi$,

② is $\quad -n^2 + s^2 - 2nsi + \frac{\gamma}{m}\cdot n\cdot i - \frac{\gamma}{m}\cdot s + \frac{k}{m}$

$\therefore \; -n^2 + s^2 - \frac{\gamma}{m}\cdot s + \frac{k}{m} = 0 \quad / \quad -2ns + \frac{\gamma}{m}\cdot n = 0$

$$\therefore S = \frac{1}{2}\cdot\frac{\gamma}{m} \qquad n^2 = \frac{1}{4}\cdot\left(\frac{\gamma}{m}\right)^2 - \frac{\gamma}{m}\cdot\frac{\gamma}{2m} + \frac{k}{m}$$

$$= \frac{k}{m} - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2$$

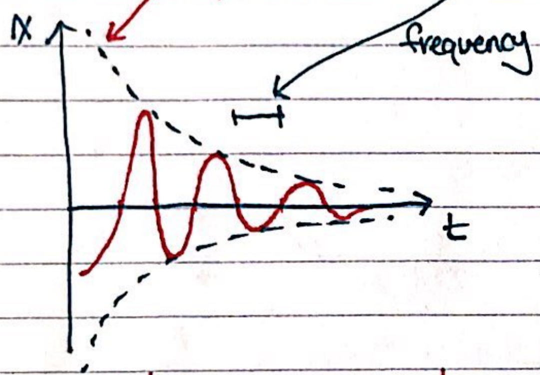
$$\therefore z = Ae^{i((n+is)t+\alpha)} = Ae^{-st}e^{i(nt+\alpha)} = \underbrace{Ae^{-\frac{1}{2}\left(\frac{\gamma}{m}\right)}}_{\text{Amplitude}}\cdot\underbrace{e^{i(nt+\alpha)}}_{\text{frequency}}$$

$$\frac{k}{m} = w_0^2, \quad n^2 = \frac{k}{m} - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2 \qquad \text{let's } n^2 = w^2 = w_0^2 - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2$$

$$w = \sqrt{w_0^2 - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2}$$

$$z = A^{-\frac{1}{2}\left(\frac{\gamma}{m}\right)t}\cdot e^{i(wt+\alpha)}$$

$$x = \underbrace{Ae^{-\frac{1}{2}\left(\frac{\gamma}{m}\right)t}}_{\text{amplitude}}\cdot\cos(wt+\alpha)$$



as t increase amplitude decrease however it is not dependent on frequency.

$$T = \frac{2\pi}{w} = 2\pi\cdot\frac{1}{\sqrt{w_0^2 - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2}}$$

c) $m\ddot{x}+\gamma\dot{x}+kx=e^{i\omega t}$, for steady state solution.

Let's $\frac{k}{m}=\omega_0^2$, $z=Ae^{i(\omega t-\alpha)}$

$$Ae^{i(\omega t-\alpha)}\left(-\omega^2\cdot m+\omega\gamma i+k\right)=e^{i\omega t}$$

$$A\left(-\omega^2+\frac{1}{m}\omega\gamma i+\omega_0^2\right)=\frac{1}{m}e^{i\alpha}=\frac{1}{m}(\cos\alpha+\sin\alpha\, i)$$

$A(-\omega^2+\omega_0^2)=\frac{1}{m}\cos\alpha$ ... ①

$A(\frac{1}{m}\omega\gamma)=\frac{1}{m}\sin\alpha$ ... ②

$\rightarrow \tan\alpha=\dfrac{\frac{1}{m}\omega\gamma}{-\omega^2-\omega_0^2}$

①^2+②^2 $\quad A^2\left\{(-\omega^2+\omega_0^2)^2+(\frac{1}{m}\omega\gamma)^2\right\}=\frac{1}{m^2}$

$$\therefore A=\frac{\frac{1}{m}}{\sqrt{(\omega_0^2-\omega^2)^2+(\frac{\omega\gamma}{m})^2}}$$

$\therefore z=Ae^{i(\omega t-\alpha)}$, $x=A\cdot\cos(\omega t-\alpha)$

Since $m=k=1$, $\gamma=0.1$, $\frac{k}{m}=\omega_0^2$ $\therefore \omega_0=1$

$A=\dfrac{1}{\sqrt{(\omega_0^2-\omega^2)^2+\frac{\omega^2}{100}}}$, $x=A\cdot\cos(\omega t-\alpha)$

phase difference

$$\boxed{x=\frac{1}{\sqrt{(1-\omega^2)^2+\frac{\omega^2}{100}}}\cdot\cos(\omega t-\alpha)},\quad \boxed{\tan\alpha=\frac{\omega}{1-\omega^2}}$$

whe ω goes ∞ $A=0$, ω goes 0, $A=1$

$\alpha=0$