6.1 a) Work out the first three cumulants C_1, C_2, and C_3.

From (6.25) $\exp\left(\sum_1^\infty \frac{(ik)^n}{n!} C_n\right) = \sum_0^\infty \frac{(ik)^n}{n!} \langle x^n \rangle$

(6.26) $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \cdots$

$$e^{(ikC_1 - \frac{k^2}{2}C_2 - i\frac{k^3}{6}C_3 + \cdots)} = \langle x^0 \rangle + ik\langle x^1 \rangle + \left(\frac{-k^2}{2}\right)\langle x^2 \rangle - \frac{ik^3}{6}\langle x^3 \rangle \cdots$$

$\rightarrow X = (ikC_1 - \frac{k^2}{2}C_2 - i\frac{k^3}{6}C_3 \cdots)$ to 6.26

$$1 + X + \frac{X^2}{2} + \frac{X^3}{6} + \cdots = \langle x^0 \rangle + \underline{ik\langle x \rangle} + \underline{\left(\frac{k^2}{2}\right)\langle x^2 \rangle} - \underline{\frac{ik^3}{6}\langle x^3 \rangle} \cdots$$

$$1 + (ikC_1 - \frac{k^2}{2}C_2 - \frac{k^3}{6}iC_3 \cdots) + \frac{1}{2}(-k^2C_1^2 - \cancel{2}k^3 i C_1C_2 \cdots) + \frac{1}{6}(-ik^3C_1^3 \cdots$$

$\therefore 1 = \langle x^0 \rangle$

$\cancel{ik}C_1 = \cancel{ik}\langle x^1 \rangle \longrightarrow \boxed{C_1 = \langle x \rangle}$

$-\frac{\cancel{k^2}}{\cancel{2}}C_2 - \frac{1}{2}k^2C_1^2 = -\frac{\cancel{k^2}}{\cancel{2}}\langle x^2 \rangle \quad \boxed{C_2 + C_1^2 = \langle x^2 \rangle}$

$-\frac{k^3}{\cancel{6}}iC_3 - \frac{k^3}{2}iC_1C_2 + \frac{-1}{6}ik^3C_1^3 = -\frac{\cancel{ik^3}}{\cancel{6}}\langle x^3 \rangle$

$\boxed{C_3 + 3C_1C_2 + C_1^3 = \langle x^3 \rangle}$

$C_1 = \langle x \rangle$

$C_2 = \langle x^2 \rangle - \langle x \rangle^2$

$C_3 = \langle x^3 \rangle - 3\langle x \rangle(\langle x^2 \rangle - \langle x \rangle^2) + \cancel{\ } \langle x \rangle^3$

$= \langle x^3 \rangle - 3\langle x \rangle\langle x^2 \rangle + \cancel{\$}\langle x \rangle^3$