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Designing One-DOF Mechanisms for Architecture by Rationalizing Curved Folding

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Abstract

We propose a modeling method based on rationalizing curved folding in order to find the form variations of 1DOF origami mechanism. We interact with a physical paper model of curved folding and then discretize a curved folding by identifying and fixing the rulings. The discretized form is a rigid origami structure with at most one degree of freedom. The form adjustment follows the discretization so that it is sure to realize a mechanism. The workshop performed by the authors based on the proposed design method is reported. The objective of the workshop was to utilize the 1DOF characteristic of discretized curved folds as a constraint in the design of dynamic architectural components. The results showed the feasibility of the method and suggested a novel methodology for designing.

Keywords: kinetic architecture, curved folding, rigid origami, form finding, physics-based design

1 Introduction

1.1 Kinetic Surface using Rigid Origami

Kinetic design in architecture is very important, e.g., dynamic solar shading, doors and openings, acoustic controllers, or portable furniture; virtually any architecture has kinetic components to be functional. However, the design of such components is morphologically poorly investigated field. This can be attributed in part due the challenge imposed by an extra dimension (time) apparent in the component design. Our objective is to add more design freedom in such components so that we can explore variations of multi-functional, context-matching, high-performance, efficient, and super cool-looking designs.

In order to design such a kinetic structure in a relatively big scale, it is useful to use geometric method that requires no or few material deformation in order to achieve the overall transformation. Two major geometric approaches for achieving mechanical structures are 1) to use bars and rotational hinges to construct scissors mechanism, e.g., the works of Santiago Calatrava and Chuck Hoberman; and 2) to use plates and hinges to form polyhedral surface linkage with a synchronized motion, i.e. rigid origami. In this paper, we focus on the designs of the latter, since they have an advantage over the bars and hinges mechanisms in the sense that they can form a kinetic watertight surface; a watertight surface is suitable for partitioning architectural spaces. However, unlike the bar linkage models, the form of rigid origami structures is heavily constrained, and it is has been very hard to find really useful design solutions.

There have been studies of rigid origami for forming kinetic surface such as Miura-ori [1], Ron Resch's Pattern [2], and Chuck Hoberman's designs [3]. Especially, mechanisms based on a quadrilateral mesh such as Miura-ori produce 1DOF synchronized motions. This implies that the whole structure transforms by asserting force to a part of the structure. This is very useful for designing kinetic architectural components because of the following reasons.

1. The structure can be formed by thick panels and hinges and thus is scalable to architectural structure.
2. The overall structure is controlled by one manipulator or a human user.
3. It is possible to design the structure that counterbalances itself in the transformation.
4. The structural mechanism is redundant and we can cut out holes to surface while preserving the mechanism.

Recent studies generalize theses structures to achieve further design freedom. Tachi [4][5] proposes computational design methods to obtain variations of miura-ori-like quadrilateral mesh structures while preserving their flat-foldability and rigid-foldability. However, since this method uses continuous transformation from one pattern to another, the achievable forms are restricted by the initial patterns, the finding of which is still an important yet unsolved problem.

1.2 Contributions

In order to find the form variations of 1DOF rigid origami, we propose a modeling method that combines physical curved folding and computational design method. We have performed a workshop [6] based on this approach. The results demonstrated that the method is feasible and useful. This also suggests a new paradigm of design methodology. The following is the summary of contribution.

1. The method includes physical interaction with paper in the brainstorming stage. The design process starts from physical interaction with paper by cutting, bending, folding and curved folding. This allows designers to explore wide variety of possibilities and to be inspired from the reaction of the physical paper.
2. Rigid origami interpretation of curved folding enables the flexible designs of mechanisms. We make 1DOF mechanisms by identifying the rulings using a physical approximation method, and linking adjacent vertices in order to rationalize curved folded paper as a planar quadrangle mesh.
3. The workshop results demonstrate the feasibility and usefulness of our proposed approach as a design method. They also suggest a novel methodology to design that results from embedding understanding of the connection between physical material, geometry, and architectural applications within a predefined process.

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2 Design Method

2.1 Method Overview

The proposing design process is as follows:

1) We first start from interacting with a physical model of curved folding made from paper or unwoven fabric, which can change its shape by flexibly adapting its rulings position.
2) Once we get some meaningful form, we discretize a curved folding by identifying and fixing the rulings. The discretized form is a rigid origami structure with at most one degree of freedom.
3) We use Freeform Origami and Rigid Origami Simulator [7] to simulate folding of the resulting discretized pattern and solve the developability and planarity constraints. We add other geometric constraints consistent with the design problem, and adjust the patterns to fit the design intention.
4) We implement the surface designs as physical mechanisms using thin sheet or thick panels cut by vinyl cutter, laser cutter, or other 3-axis machines.

2.2 Interaction with Curved folding

Curved folding is a piece of paper folded along curved creases; it is formed by combining folding and bending of the paper. In a geometric sense, a curved folded shape is a developable surface comprising the patches of smooth (G^2 continuous) developable surface: plane, generalized cylinder, tangent surface, and generalized cone. Even though each of these smooth patches only makes relatively simple form, the existence of creases make the surface freely configurable, enabling globally non-developable complex surface, e.g., approximated hyperbolic surface and elliptic surfaces [8], the sculptures by David Huffman [9], the early computer graphics works of Ron Resch [10], and product designs by Gregory Epps (Fig. 1-2).

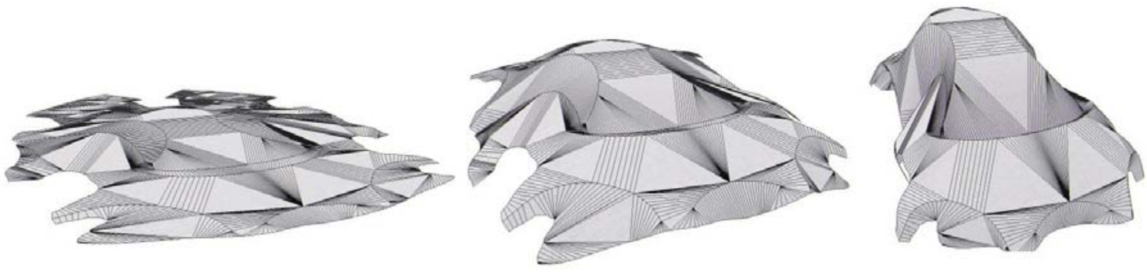

Fig. 1: A car model by Gregory Epps animated by Rigid Origami Simulator [7], using the rulings reconstruction by Killian et al. [15]

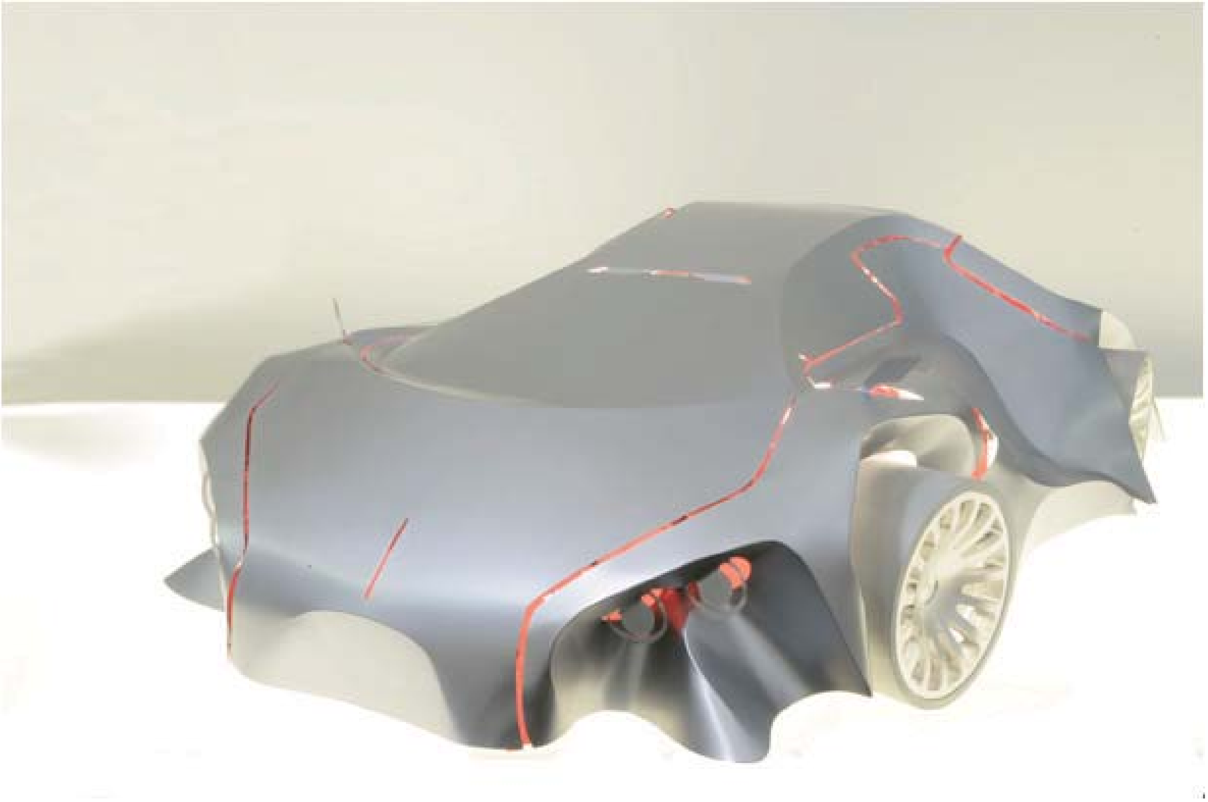

Fig. 2: A car model by Gregory Epps.

Each smooth developable surface patch is a ruled surface, where the rulings have non-twisting relationship with neighborhood (this is because of the developability). A state of curved folding is parameterized by the position of creases and rulings on the surface, the rotational angle at the crease, and the maximum curvature of the surface at every point (perpendicular to the ruling passing through the point).

The differential geometric analyses of curved folding by Huffman [11] and by Fuchs and Tabachnikov [12] give the basic understanding of how a deformation of a curved fold such as folding and twisting affects the change in the rulings and the folding angles. The following results from [12] are useful to understand the relationship between the three-dimensional configuration and the crease pattern (ruling pattern) of a curved fold.

1. The curvature of a fold line in space increases as it folds. The relationship is proportional: $k_{3D} = k_{2D} / \cos\alpha$, where $k_{3\mathrm{D}}$ and $k_{2\mathrm{D}}$ are the curvature (the inverse of the radius of the curvature) of the foldline in the space and the crease pattern respectively; α is the half of the folding angle of the crease at the point. From this fact, we can understand that a straight line in the crease pattern cannot be bent and is kept straight in the space until it is folded 180 degrees (Fig. 3).
2. The curved fold does not twist when the left and right rulings have the same orientation in the crease pattern. The curved fold is twisted when the rulings have kinks at the rulings (Fig. 4).

3. The folding angle, i.e., complementary angle of dihedral angle, along a curved fold is constant when the rulings reflect at the curved fold in the crease pattern, producing uniform curved folding (Fig. 5).

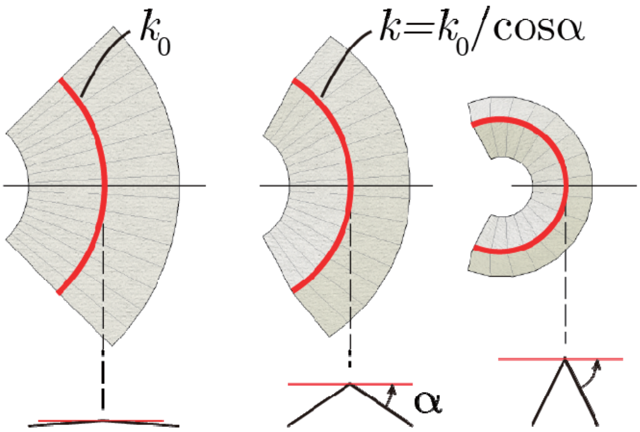

Fig. 3: Pattern-3D Relation 1: Curvature and fold angle.

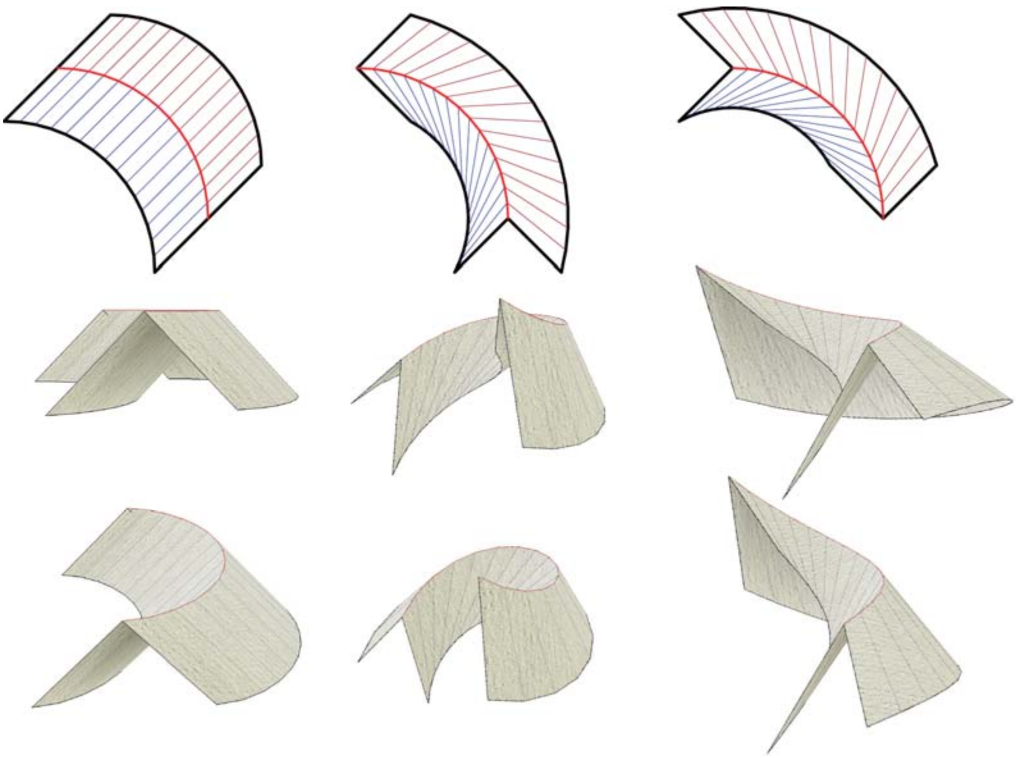

Fig. 4: Pattern-3D Relation 2: Rulings and twisting. Left: Non-twisting curved folding by straight rulings alignment in the crease pattern. Middle & Right: Twisting curved folding by kinked rulings alignment.

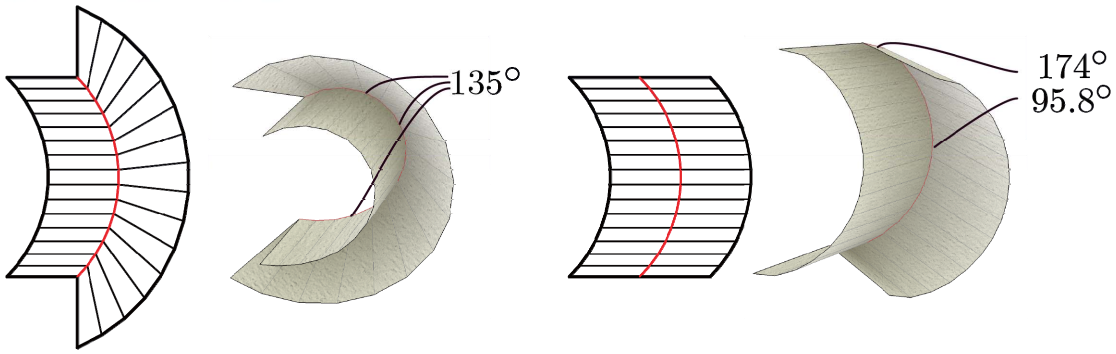

Fig. 5: Pattern-3D Relation 3: Uniform and non-uniform folding. Left: Uniform angle folding using mirror reflecting rulings. Right: Non-uniform folding (generic). The folding angle is not constant along the crease.

Result 2 implies that a non-twisting curved folding is composed of a developable surface and its mirror reflection with respect to the plane that includes the curved fold. It corresponds to the known method for creating a curved folding by cutting and mirror reflecting a single developable surface (Fig. 6). However, this is only a limited case, and we can find more form variations with twisted curved creases by physical interaction with paper.

Normally, a piece of paper deforms plastically and thus irreversibly by folding, and it does elastically thus reversibly by bending. Here, folding implies a local rotation around fixed crease, while bending includes, in addition to the change in the absolute amount of curvature (distributed rotation along the rulings), the position and orientation of rulings (Fig. 7). Therefore, a physical interaction with a curved folding implies more degrees of freedom than a regular straight line origami, where the configuration is basically represented only by the folding angles.

Using the difference in the behavior, we can design patterns in the following two steps. First, we fold the paper to set the crease shape to design overall approximate form. Then, by reversibly and continuously bending the folded shape, we can find twisted or non-uniformly transforming variations. An advantage of this physical method is that we can interactively find twisted variations of the same curved folding pattern with different ruling alignment. The form is unstable in this stage, but the form can be fixed in the rationalization step described in the next section since the twisting motion is recorded as the ruling position on the surface.

Since creasing paper or card is irreversible, it is naturally required that the designer makes lots of test models. When a more fine adjustment of crease shape is required, we can use unwoven fabric such as felt instead of paper, so that we can achieve reversible folding. This adds further degrees of freedom of designs by allowing a designer to interactively shift the crease position in the pattern, rather than defining the crease pattern in the developed state.

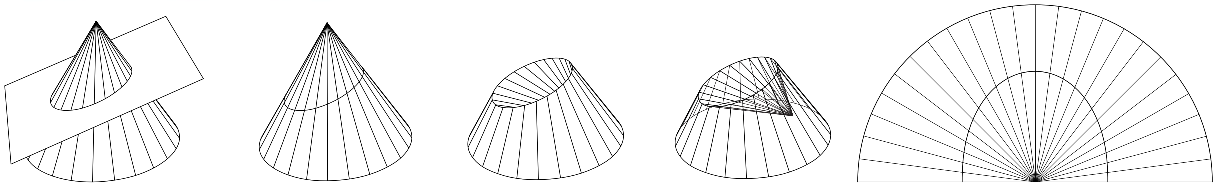

Fig. 6: Curved folding design using mirror reflection of a developable surface (*a more elaborate design example is Huffman's cone design: see* [9]).

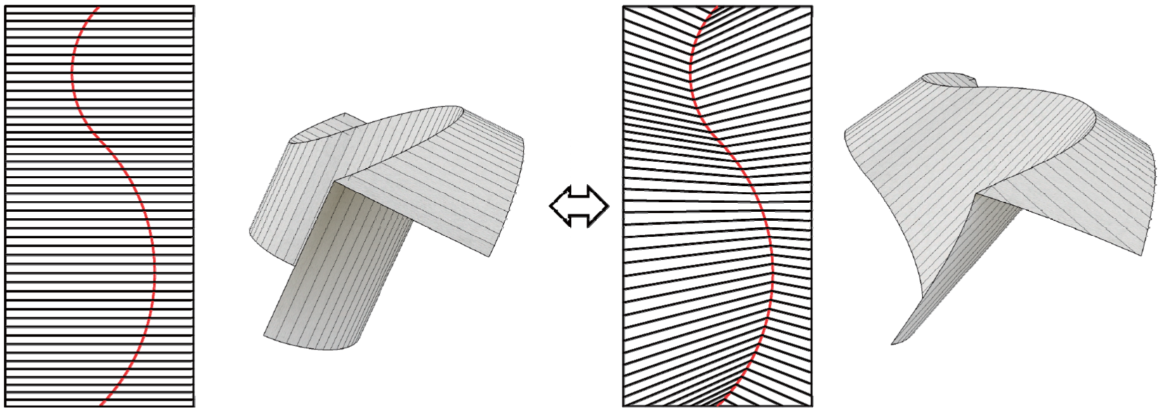

Fig. 7: The elastic transformation with the change in the rulings position (Bending only).

2.3 Rigid Curved Folding

By discretely sampling the rulings, we can represent developable surfaces and hence curved folded surfaces as quadrangle meshes. The non-twisting property of developable surface forces the adjacent rulings to form a planar quadrangle. Thus a curved folded surface is approximated by a planar-quadrilateral (PQ) mesh (as this representation is used in Figs. 2-6). PQ mesh representation of developable surface is used to simulate the bending and creasing of paper by Kergosien et al. [13], and is also studied by Liu et al. [14]. A computational method to reconstruct a PQ mesh from a laser-scanned curved folding is proposed by Kilian et al. [15]. We use much simpler method for obtaining the PQ mesh from the physical model: aligning a straightedge on the surface to find the rulings, and then drawing lines along them. We unfold the surface and draw the developable PQ mesh as a straight graph in the plane, i.e., crease pattern. The smooth curvature of the fold line is also rationalized by a series of straight lines (polyline) that link the ends of adjacent rulings, thus completing the quad facet, and therefore rationalizing both the bending and folding,

We interpret such an obtained PQ mesh as a rigid origami structure i.e., mechanism by rigid facets connected by rotational hinges. Such a structure has less flexibility than the original physical curved folding since the change in the position and orientation of a ruling (now discretized as a crease) is not allowed in the rigid origami structures. Once the surface configuration is recorded as a crease and ruling pattern in 2D, the form that can be produced from the pattern with the rigid origami motion is no longer flexible. They try to reproduce the exact 3D form that we designed through physical interaction. Therefore, rigid origami interpretation of curved folding encodes 3D and 4D forms into 2D pattern. Interpreting the 2D as 3D or 4D is a simple matter of defining an angle constraint between any two faces.

A PQ mesh rigid origami structure is either a 1DOF mechanism or an over-constrained static structure. In generic cases, the structure is not rigid-foldable, i.e., there exists no mechanism. This is because each interior vertex composed of four edges is already a 1DOF mechanism, in which one of the folding angles determines the entire configuration; if we align these vertices, the motion of every edge is multiply defined by all incident edges (Fig. 8). Rigid foldability of the PQ mesh is ensured in these known special cases, but the general condition is still an open problem:

1. When the model is representing one curved crease ($2 \times n$ array of facets) or the structure essentially repeats single curved crease.
2. When the similar vertex structure is repeated to avoid contradiction.
3. When the crease pattern is flat-foldable and there is at least one 3D state (as shown in [4]). Flat-foldable pattern is a discrete version of reflecting rulings that produces uniform folding.

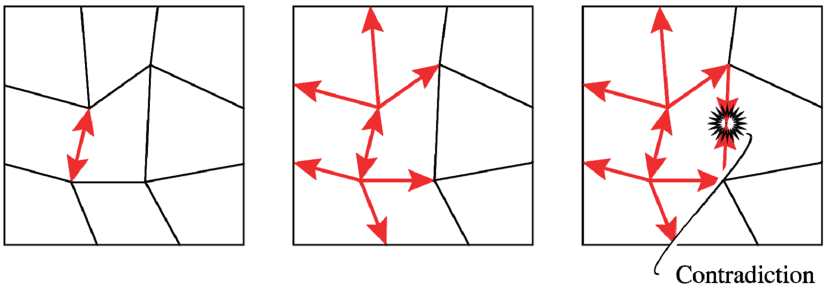

Fig. 8: Contradiction of fold angles in quadrangle mesh rigid origami.

2.4 Adjustment

We adjust the patterns so that they achieve certain properties that suit the design problems. In order to keep the developability while we adjust and add extra geometric constraints, we utilize Freeform Origami [8]. Freeform Origami is the freely available software which enables interactive pattern variations under several geometric constraints related to origami. Using the software, rigid foldable structures and bi-stable structures can be designed from a PQ mesh. These patterns can also fit pre-existing geometric conditions given by the design contexts.

Bi-Stable Structure

Even though the reconstructed pattern does not always produce the rigid folding motion, in most cases, we can at least make a bi-stable structure that transforms from a planar state to a three-dimensional state with a small amount of elastic bending of the sheets. This is because 1) most patterns that represent curved folding in 3D are continuously rigid foldable if we add a triangulation crease to emulate small amount of bending of the panel; and 2) since there is an original physical model, we can expect that there is at least one 3D configuration that is valid even without triangulation. We can obtain a bi-stable folding structure by the following procedure in the software:

1) Fold the mesh in the simulation mode, in which the system simulates rigid origami motion numerically solving unstable truss model, while allowing triangulation. We

obtain close enough shape to the 3D configuration used for obtaining the PQ mesh.

2) Switch to design mode and add developability constraints to ensure it is made from a sheet, and planarity constraints to forbid triangulation. This adjusts the pattern to be a valid 3D configuration without triangulation.
3) Set additional geometric conditions for the target 3D shape, such as the fixed shape of the pattern in the developed state and the geometric attachment to existing buildings.

In this way, we can build a valid 3D model that can also exist in a developed state. Since the transformation from the developed state to the 3D state is not generally ensured, we obtain a snap-through mechanism that is both stable in the developed and 3D states.

Rigid Foldable Structure

In the case where the pattern is close to locally flat-foldable (uniform curved folding), we can configure the system to force the pattern to be flat-foldable in the step 2) of the procedure. If the pattern converges to one three-dimensional configuration, the rigid foldability is ensured. Such mechanism has exactly one degree of freedom.

If the pattern is not close enough to flat-foldable, adding triangulation creases is a general way to turn bi-stable structure into a rigid foldable structure. If every facet is triangulated, a generic pattern has N-3 degrees of freedom where N is the number of edges on the perimeter. We can experimentally increase and decrease the number of creases until we get 1DOF rigid-foldable structure.

2.5 Implementation

Rigid origami can be formed by thick panels and hinges geometrically. This allows us to scale up the structure to architectural size. A general and effective method for thickening is to use two layers of constant thickness panel sandwiching ideal zero-thickness hinge material [5]. The essence of the idea is to make a thick groove along the edge at the valley side of the panel to avoid intersection when folded along the hinge in the center (Fig. 9). Each panel can be modeled by cutting out a two-dimensional polygon, thus can be manufactured using 3-axis CNC machines (where the third axis is for the depth of cut) such as cutting plotter, laser cutter, and milling machines. We can place piano hinges at the shared edges of the panels to create the connection. Also, if we place fabric between the layers and glue them together, the fabric acts as rotational hinges between the panels.

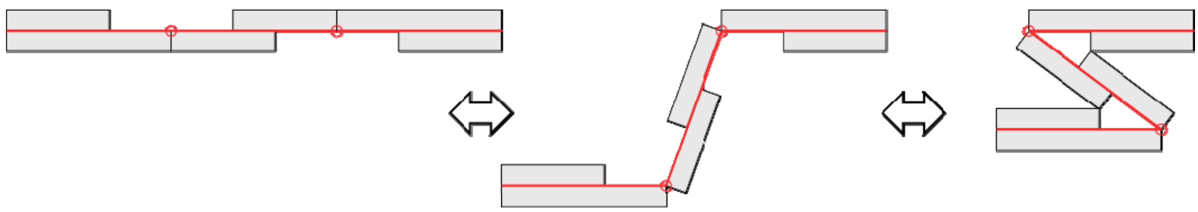

Fig. 9: Implementing rigid origami with thick panels.

3 Workshop

The authors have performed a workshop [7] based on our proposed design method. The objective of the workshop was to utilize the 1DOF characteristic of discretized curved folds as a constraint in the design of dynamic architectural components. Curved folded components demonstrate a continuous transformation of states from flat to folded, and so are ideal for openings in architectural envelopes – in this case doors and solar shading were identified as suitable examples. Participants in the workshop developed a tacit understanding of curved folds through extensive paper folding exercises (Stage 1), and then applied this knowledge to the workshop briefs; 'Doors and Openings' and 'Dynamic Solar Shading' (Stage 2), before learning how to manually identify rulings on the surface in order to translate their designs into a PQ mesh representation for manipulation and folding simulation in Freeform Origami [6] and Grasshopper [16] (Stage3). The final designs were then printed and re-built using foam-core facets and paper or tape hinges, and assembled into the desired configuration (Stage 4).

The fact that the workshop operates as one continuous design process, and each step further informs the designs of the next step, allowed for continuity of thinking by participants and allowed for amplified results at the multiple levels that this design methodology operates on. By employing what could be called a *physically responsive process based design methodology*, whereby formal aspects are allowed to develop stepwise from imposed limitations such as material and geometry, explicit outcomes are unknown, but the parameters within which they occur are implicitly understood. This differs from a typical design process that involves thinking of a design mentally and visually, and working back from this to understand manufacturability, functional suitability etc. This would again suggest that the designs are limited – but to the contrary, the results demonstrate unexpectedly elegant solutions to the design briefs.

Here, we elucidate on the process in Stages 1 and 2, since they are most novel in their approach.

3.1 Abstracting Design Intent

The primary purpose of Stage 1 is as a method of abstraction to encourage participants to be informed by the medium, rather than their expectations of form generation – this is due to the unpredictability of geometric possibility in curved folding, when compared to the current modeling paradigm, which could be considered as a clay-modeling paradigm.

Abstracting Briefs

The class therefore starts with a number of abstract briefs that refer to effects and procedures in paper folding, rather than architectural program: change size, strengthen, articulate, encase, protect, join things and store energy Participants were required to demonstrate successful meeting of this brief through multiple models (Fig. 10). During modeling, further constraints were added in order to continuously force abstraction, such as using algorithmic approaches [17][18] and breaking up chunks of time into short ten minute exercises to enlarge, divide, join and multiply as mechanisms to change the geometry using physical methods, rather than relying on mental imagery to design.

Common Library

The outcome of the introductory design stage, as well as familiarizing the participants with the methods of design that are effective for the curved folding medium, was to create a shared library of geometries that represent the investigative areas that one could encounter when designing in this medium. Each group placed their work on a common wall, building a common toolkit of shape typologies encapsulating the fast paced learning for shared use in the main design session (Fig. 11). The physical and visual array of form, mechanisms, perforation techniques, articulating actions, and approaches provided a simple storage and retrieval system for the design ideas during the two day workshop, as it remained on display for reference. Participants were also required to share their experience and beliefs about their favorite work with the class.

3.2 Applied Briefs

As workshop leader of the design phase, Gregory Epps aided in making a down-selection from the available media, identifying appropriate geometries for the participants to further pursue. With the brief of "Dynamic Solar Shading" (Fig. 12-13) and "Doors and Openings" (Fig. 14-15) now active, groups of four to seven focused on refining articulation, function and form by increasing scale and adding broader levels of detail as they progressed, such a fixing locations, methods of actuation, hinge and material types and aesthetics. Because the whole group had distributed the task of researching and disseminating these core aspects, their ability to express themselves was evident within the timeframe of the two day workshop (Fig. 10-15).

Further Improvements

In further iterations of this teaching technique, it is suggested the knowledge gained in Stage 1 is explicitly spread through to the Stage 2 by exploding the initial teams from the introductory session, and using one member form each team to build the teams that will apply the knowledge.

4 Conclusion

We presented a novel method for designing rigid foldable structure through physical interaction with paper to create a curved folding and then rationalizing the folded surface. The combination of physical interaction and digital/geometric manipulation allows the designers to understand and to discover the connection between the design material, geometry, and the potential application in architecture. Our approach can be compared to completely geometric and generative approach in which it is too restrictive to explore new ideas for designs, or purely physical approach in which we cannot really understand the 4D behavior of the structure when it is really build with panels and hinges. This approach led the success in the design workshop.

The workshop posed multiple challenges for participants on a geometric level – they had to learn how to conceive physically, rather than mentally, their thoughts on design of a 4D object, using a surface representation that differs hugely for the standard modeling paradigms, and transpose these forms to a secondary linkage based system, that could be argued is very distant from the initial medium of curved folding, and had to meet strict geometric requirements for this latter stage to function effectively. Contrary to these limitations, the *physically responsive process based design methodology* increases variation in aesthetics and function of the output.

Fig. 10: Stage 1: Design Abstraction; articulation experimentation of crumpled paper

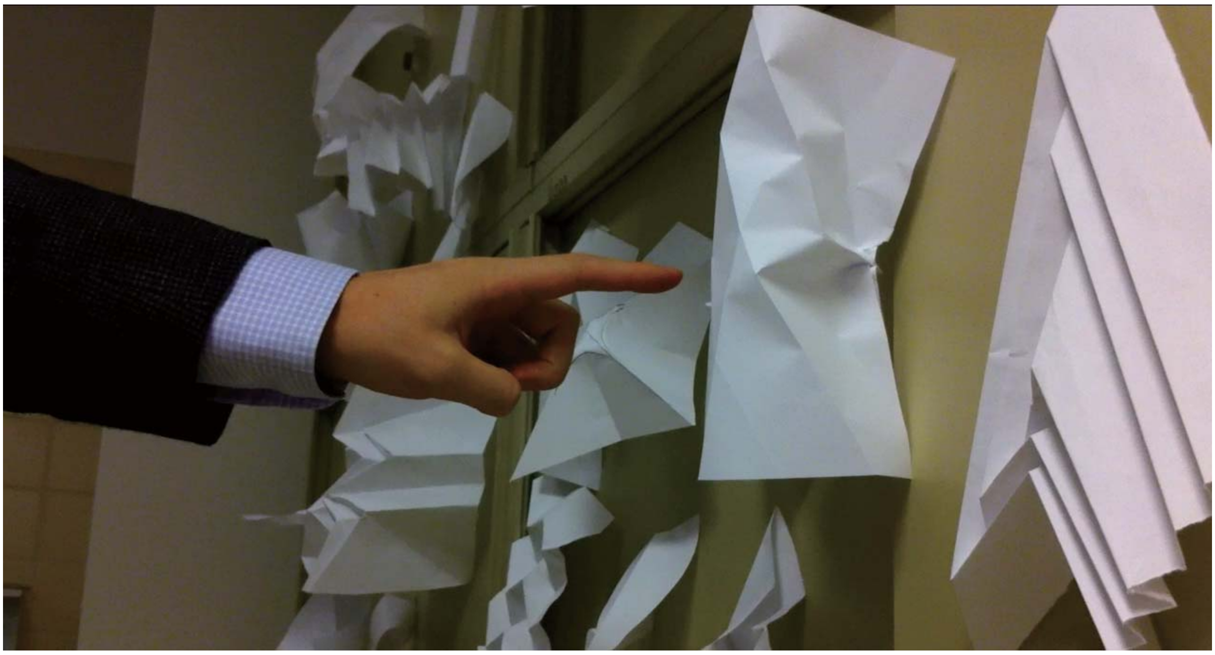

Fig 11: Stage 1. Common Library (Shape Typology Toolkit); review of all participants

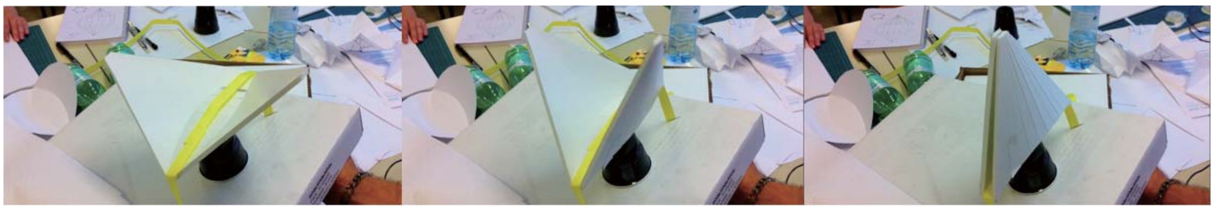

Fig 12: Stage 2. Group Brief: Dynamic Solar Shading (members: Emilie Hergott, Nicolas, Leduc, Sophie Pennetier, and Simon Schleicher); upward motion actuated by downward force on exterior edge

Fig 13: Stage 2. Group Brief: Dynamic Solar Shading; hexagonal array of articulated shading elements

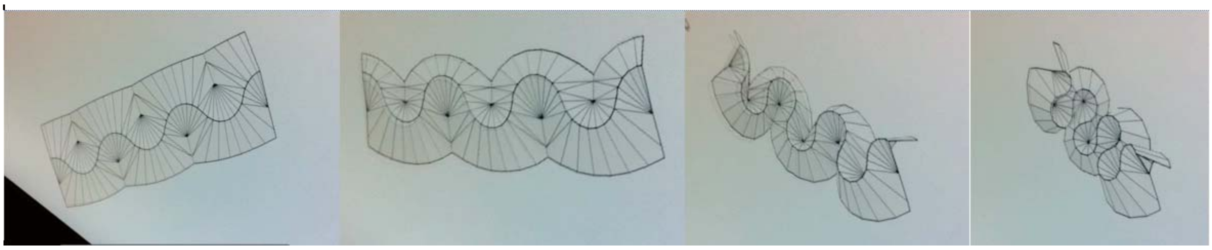

Fig. 14:Stage 2. Group Brief: Doors and Openings (members: Jacek Jaskólski, Thorsten Lang, Ana Maria Marcu, Ioanna Symeonidou, and Liam Woolley); animation in Rigid Origami Simulator. Notice the triangulation added to enable a mechanism.

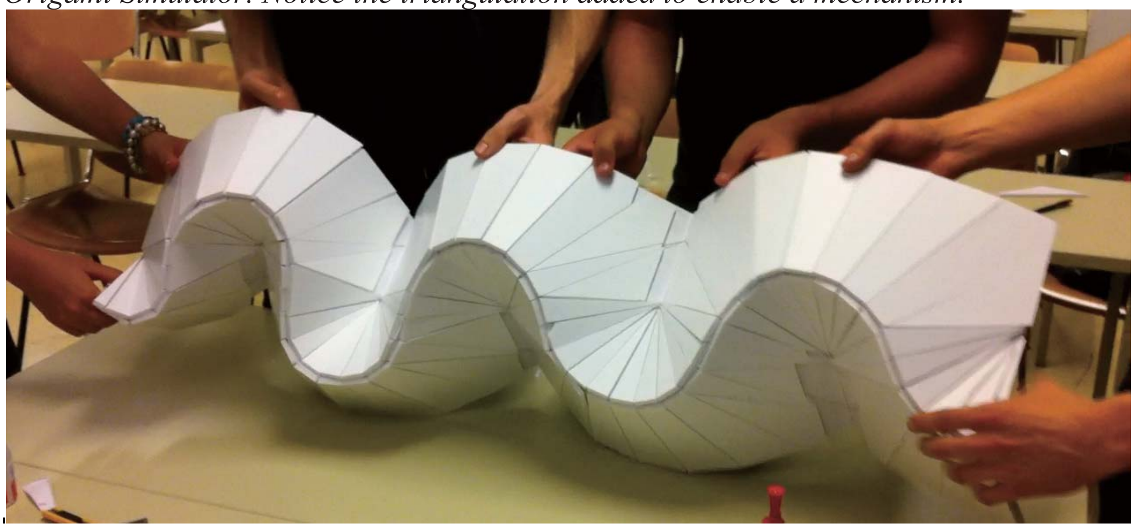

Fig. 15: Stage 2. Group Brief: Doors and Openings; two layer rigid plate model sandwiching a paper hinge, pattern generated using Grasshopper [16] definition by Tomohiro Tachi; assembly according to [5]

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